UNIVERSIDADE TECNOLÓGICA FEDERAL DO PARANÁ
GRADUATE PROGRAM IN ELECTRICAL ENGINEERING

ANDRÉA MACARIO BARROS

**A FAST GRIDLESS ALGORITHM FOR HARMONICS AND
INTERHARMONICS ESTIMATION**

DISSERTATION

PATO BRANCO
2020

ANDRÉA MACARIO BARROS

**A FAST GRIDLESS ALGORITHM FOR HARMONICS AND
INTERHARMONICS ESTIMATION**

Dissertation presented to Graduate Program in Electrical Engineering of Universidade Tecnológica Federal do Paraná – campus Pato Branco – as a partial requirement to obtain the title of Master in Electrical Engineering.

Supervisor: Prof. Dr. Giovanni Alfredo Guarneri

**PATO BRANCO
2020**

B277f Barros, Andréa Macario.
 A fast gridless algorithm for harmonics and interharmonics estimation /
 Andréa Macario Barros. -- 2020.
 77 f. : il.

 Orientador: Prof. Dr. Giovanni Alfredo Guarneri
 Dissertação (Mestrado) - Universidade Tecnológica Federal do Paraná.
 Programa de Pós-Graduação em Engenharia Elétrica. Pato Branco, PR,
 2020.
 Inclui bibliografia.

 1. Energia elétrica. 2. Energia elétrica distribuição. I. Guarneri, Giovanni
 Alfredo, orient. II. Universidade Tecnológica Federal do Paraná. Programa
 de Pós-Graduação em Engenharia Elétrica. III. Título.

CDD 22. ed. 621.3

Ficha Catalográfica elaborada por
Maria Juçara Vieira da silveira CRB9/1359
Biblioteca da UTFPR Campus Pato Branco

TERMO DE APROVAÇÃO DE DISSERTAÇÃO Nº 90

A Dissertação de Mestrado intitulada "***A Fast Gridless Algorithm for Harmonics and Interharmonics Estimation***", defendida em sessão pública pela candidata **Andréa Macario Barros**, no dia 02 de outubro de 2020, foi julgada para a obtenção do título de Mestre em Engenharia Elétrica, área de concentração Sistemas e Processamento de Energia, linha de pesquisa Análise de Sistemas Dinâmicos, e aprovada em sua forma final, pelo Programa de Pós-Graduação em Engenharia Elétrica.

Banca Examinadora:

Prof. Dr. Giovanni Alfredo Guarneri - UTFPR/PB (Presidente da Banca);
--

Prof. Dr. Fábio Luiz Bertotti - UTFPR/PB;
--

Prof. Dr. André Eugênio Lazzaretti - UTFPR/CT;

Prof. Dr. Miguel Moreto - UFSC/SC.

Este Termo de Aprovação encontra-se na pasta da aluna na Secretaria do Programa, contendo a assinatura da Coordenação realizada após a entrega da versão final, incluindo correções necessárias, permitindo o encaminhamento para análise e publicação no Repositório Institucional.

Assinado eletronicamente por

Gustavo Weber Denardin

Coordenador do Programa de Pós-Graduação em Engenharia Elétrica

AGRADECIMENTOS

Primeiramente gostaria de agradecer a Deus por todas as bênçãos que me concedeu e me concede a cada dia que passa. Sempre ao meu lado e base para todos os momentos.

Gostaria de agradecer ao meu orientador, professor Giovanni Alfredo Guarneri, por me conceder a honra de ser sua orientanda e por sua atenção, paciência e valiosas orientações. Agradeço a Coordenação de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) pelo financiamento que permitiu minha dedicação exclusiva a essa pesquisa.

Agradeço também aos meus pais André Macario Barros e Izabel Cristina Barros por sempre me incentivarem a buscar conhecimento e fazerem o que é possível para me ajudar a seguir meus sonhos, quem sou devo a vocês! Agradeço ao meu irmão Gabriel Macario Waidzk Barros por seu meu melhor amigo e meu companheirinho a todos os momentos. Agradeço ao meu avô Antônio Souza Barros pelo infinito amor e carinho e por me mostrar que podemos conquistar o mundo.

Agradeço aos meus colegas e amigos de laboratório Andréia, Marco e Tatiana. Andréia, pelos anos de amizade, aventuras, conversas e companheirismo. Marco por todas risadas e ideias trocadas, as quais foram valiosas para o desenvolvimento desse trabalho. Tatiana agradeço imensamente pela sua dedicação e organização, seu trabalho foi base para meus estudos e desenvolvimento dessa dissertação.

Por fim, agradeço a todos meus bons amigos e a todos de que algum modo colaboraram para essa caminhada.

"Eu sou Moana de Motu Nui. A bordo do meu barco, navegarei por todo o mar e restaurarei o coração de Te Fiti."
Moana

ABSTRACT

BARROS, Andréa M.. A FAST GRIDLESS ALGORITHM FOR HARMONICS AND INTER-HARMONICS ESTIMATION. 77 f. Dissertation – Graduate Program in Electrical Engineering, Universidade Tecnológica Federal do Paraná. Pato Branco, 2020.

The increase of electronic power devices connected to the electric power system may cause the addition of harmonics and interharmonics components in the system signals, which may cause several adverse effects. The study and development of algorithms to correctly estimate the components that compound the power system signals considering the robustness to noise and frequency deviation are essential to guarantee the power quality of the system. In this work, a new algorithm called *Anti-Leakage Matching Pursuit* (ALMP) is proposed for estimating the harmonics and interharmonics components, avoiding the *Spectral Leakage* effect, in contrast to some techniques in the literature such as the Discrete Fourier Transform (DFT) and Harmonics and Interharmonics components Estimation based on Signal Sparse Decomposition (HIESSD). The algorithm is based on the *Matching Pursuit* technique and aims to decompose any signal into a linear combination of previously known waveforms. The main stage of the ALMP is the solution of a non-linear least squares problem by an optimization algorithm, where the waveform that best matches the signal is determined. The study of optimization techniques is carried out to determine which one will be used in the algorithm, and the most recent algorithms for harmonics and interharmonics estimation are presented. The proposed method is compared with DFT, Matrix Pencil Method (MPM), Fast Matching Pursuit (FMP), and HIESSD techniques to assess its proper estimation. In terms of amplitude, the proposed algorithm obtained results equivalent to the existing algorithms presenting a Mean Absolute Error (MAE) of 1.6×10^{-3} pu for harmonics estimation, while the DFT, MPM, FMP and HIESSD presented average MAEs of 1.8×10^{-3} pu, 4.3×10^{-3} pu, 8.3×10^{-3} pu and 8.3×10^{-3} pu, respectively, considering the same conditions. It also proved to be robust to frequency deviation, in contrast to HIESSD and DFT and presented a reduced execution time compared to the other presented algorithms, up to 24 times less than the time presented by the FMP, up to 73 times less than the time presented by the MPM and up to 294 times less than the time presented by the HIESSD.

Keywords: Frequency deviation, Harmonics and interharmonics estimation, Power quality.

RESUMO

BARROS, Andréa M.. A FAST GRIDLESS ALGORITHM FOR HARMONICS AND INTER-HARMONICS ESTIMATION. 77 f. Dissertation – Graduate Program in Electrical Engineering, Universidade Tecnológica Federal do Paraná. Pato Branco, 2020.

O crescimento de dispositivos eletrônicos conectados ao sistema elétrico de potência pode provocar a adição de componentes harmônicas e interharmônicas nos sinais do sistema, o que pode causar diversos efeitos adversos. O estudo e desenvolvimento de algoritmos para a correta estimação dos componentes que compõe os sinais dos sistemas de potência considerando sua robustez ao ruído e desvio de frequência são essenciais para garantir a qualidade de energia do sistema. Neste trabalho, um novo algoritmo chamado *Anti-Leakage Matching Pursuit* (ALMP) é proposto para a estimação de harmônicos e interharmônicos, evitando os efeitos do *Vazamento Espectral*, em contraste com algumas técnicas na literatura, como DFT e HIESSD. O algoritmo é baseado na técnica de *Matching Pursuit* e procura decompor qualquer sinal em uma combinação linear de formas de onda previamente conhecidas. A etapa principal do ALMP consiste na solução de um problema de mínimos quadrados não-linear por meio de um algoritmo de otimização, nessa etapa determina-se a forma de onda que melhor se adequa ao sinal. É apresentado um estudo acerca das técnicas de otimização, afim de determinar-se qual técnica será usada pelo algoritmo, assim como os algoritmos mais recentes para estimação de harmônicos e interharmônicos são apresentados. O método proposto é comparado com as técnicas DFT, MPM, FMP e HIESSD para assegurar sua estimação apropriada. O algoritmo obteve resultados equivalentes aos dos algoritmos existentes apresentando um erro médio de 1.6×10^{-3} pu para a estimação de harmônicos, enquanto a DFT, MPM, FMP e HIESSD apresentaram erro médio de 1.8×10^{-3} pu, 4.3×10^{-3} pu, 8.3×10^{-3} pu e 8.3×10^{-3} pu, respectivamente, considerando as mesmas condições. Além disso, provou ser robusto ao desvio de frequência, ao contrário do HIESSD e DFT, e apresentou um reduzido tempo de execução quando comparado aos outros algoritmos apresentados, de até 24 vezes menor que o tempo apresentado pelo FMP, até 73 vezes menor que o tempo apresentado pelo MPM e até 294 vezes menor que o tempo apresentado pelo HIESSD.

Palavras-chave: Desvio de frequência, Estimação de harmônicos e interharmônicos, Qualidade de energia.

LIST OF FIGURES

FIGURE 1	– Simplified flowchart representing ALMP's steps.	18
FIGURE 2	– The overexcited transformer's magnetizing current signal.	20
FIGURE 3	– The single-phase DC source's input current signal.	20
FIGURE 4	– The six-pulse converter's current signal.	21
FIGURE 5	– The Cycloconverter's output signal.	22
FIGURE 6	– The Synchronous Machine's stator signal.	23
FIGURE 7	– Signal from a microgrid.	24
FIGURE 8	– Step difference between the line search and trust region approaches.	27
FIGURE 9	– Polygons considered by Nelder-Mead algorithm in $\mathbb{R}^2$ and $\mathbb{R}^3$	29
FIGURE 10	– Transformation of Nelder-Mead Algorithm.	30
FIGURE 11	– DFT Spectral Leakage.	34
FIGURE 12	– A distorted signal represented by a linear combination of atoms.	47
FIGURE 13	– Steps performed by the ALMP algorithm.	49
FIGURE 14	– Mean absolute error in estimation according to the SNR.	52
FIGURE 15	– Estimation of the amplitudes of the DC Source.	56
FIGURE 16	– Harmonics estimation for the signal of the rectifier with 40 dB.	58
FIGURE 17	– Harmonics estimation for the signal of the rectifier with 20 dB.	58
FIGURE 18	– Harmonics estimation for the signal of the rectifier with 10 dB.	59
FIGURE 19	– Spectrum obtained by the FMP algorithm considering varied noise conditions.	60
FIGURE 20	– Harmonics estimation for the signal of Synchronous Machine.	62
FIGURE 21	– Overexcited Transform Signal Decomposition.	64
FIGURE 22	– Estimation of Cycloconverter Input Signal.	65
FIGURE 23	– Estimations of the microgrid signal.	67
FIGURE 24	– Microgrid Signal Decomposition.	69

LIST OF TABLES

TABLE 1 – Comparative analysis between the optimization algorithms according to the SNR.	53
TABLE 2 – Comparative analysis between the optimization algorithms.	54
TABLE 3 – Errors presented in harmonics estimation for the signal of the rectifier.	59
TABLE 4 – MAE from the estimation of the overexcited transformer signal.	63
TABLE 5 – Execution time presented by the algorithms.	69

LIST OF ACRONYMS

DFT	Discrete Fourier Transform
HIESSD	Harmonics and Interharmonics components Estimation based on Signal Sparse Decomposition
MPM	Matrix Pencil Method
FMP	Fast Matching Pursuit
MAE	Mean Absolute Error
SSD	Sparse Signal Decomposition
FFT	Fast Fourier Transform
OMP	Orthogonal Matching Pursuit
AC	Alternating current
DC	Direct current
AC	Alternating current
QN	Quasi-Newton
SR1	Symmetric Rank-One
BHHH	Berndt–Hall–Hall–Hausman
BFGS	Broyden–Fletcher–Goldfarb–Shanno
LMA	Levenberg-Marquadt algorithm
SVD	Singular Value Decomposition
MP	Matching Pursuit
WT	Wavelet Transform
OHD	Overcomplete Hybrid Dictionary
NMA	Nelder-Mead algorithm
SNR	Signal-noise ratio

LIST OF SYMBOLS

$y(t)$	Signal
ω	Angular frequency
f	Fundamental frequency
F_s	Sampling Frequency
N	Total number of samples
i_{DC}	Charge current
h	Harmonic component
F	Objective function
$\mathbf{x}_0$	Start value
$\mathbf{p}_k$	Direction
$m_k(\cdot)$	Quadratic model
$\mathbf{B}_k$	Hessian approximation
Δ	Trust-region radius
$\mathbf{J}$	Jacobian matrix
j	Imaginary unit
ζ	Damping factor
T_s	Sampling Time
L	Pencil parameter
$\text{diag}(\cdot)$	Diagonal matrix
λ	Eigenvalues
$[\cdot]^+$	Moore-Penrose pseudo-inverse operation
$[\cdot]^H$	Conjugate transpose operation
$\text{eig}(\cdot)$	Eigenvalue calculation
Φ	Dictionary
ϕ	Atom
$\mathbf{r}$	Residue
γ	Set of variables
ν	Phase indicator
Δf	Frequency resolution
f_s	Sampling frequency
l	Fold
ψ	Wavelet mother
a	Scale parameter
b	Localization parameters
α	Vector of coefficients
ϵ	Tolerance
$\mathcal{S}$	Solution support
φ	Phase
A	Amplitude
M	Model Order
η	White Gaussian noise

CONTENTS

1 INTRODUCTION	14
1.1 OBJECTIVES	16
1.2 OUTLINE OF THE DOCUMENT	17
2 THEORETICAL FOUNDATION	18
2.1 HARMONICS AND INTERHARMONICS DISTORTIONS	18
2.1.1 Harmonics Distortions Sources	19
2.1.2 Interharmonics Distortions Sources	21
2.2 FREQUENCY DEVIATION ON POWER SYSTEMS	23
2.3 UNCONSTRAINED OPTIMIZATION	24
2.3.1 Line Search Methods	25
2.3.2 Trust-Region Methods	26
2.4 NON-LINEAR LEAST SQUARES PROBLEMS	27
2.4.1 Levenberg-Marquadt Algorithm	28
2.4.2 Nelder-Mead Algorithm	29
2.4.3 Quasi-Newton Method	31
2.5 CHAPTER REVIEW	32
3 METHODS FOR ESTIMATING FREQUENCY COMPONENTS	33
3.1 DISCRETE FOURIER TRANSFORM	33
3.2 MATRIX PENCIL METHOD	34
3.3 MATCHING PURSUIT ALGORITHM	38
3.3.1 Dictionary	38
3.4 HARMONICS AND INTERHARMONICS COMPONENTS ESTIMATION BASED ON SIGNAL SPARSE DECOMPOSITION	39
3.5 FAST MATCHING PURSUIT	42
3.6 CHAPTER REVIEW	45
4 PROPOSED METHOD	46
4.1 SIGNAL MODELING	46
4.2 DICTIONARY DESIGN	46
4.3 PARAMETERS ESTIMATION WITH ALMP	47

4.4	CHAPTER REVIEW	50
5	RESULTS AND DISCUSSIONS	51
5.1	NON-LINEAR LEAST SQUARE ALGORITHM	51
5.2	ALGORITHM EVALUATION	54
5.2.1	Single-phase DC source	55
5.2.2	Six-pulse Converter	56
5.2.3	Synchronous Machine	61
5.2.4	Overexcited Transformer	62
5.2.5	Cycloconverter	65
5.2.6	Microgrid	66
5.2.7	Execution Time	69
5.3	CHAPTER REVIEW	69
6	FINAL CONSIDERATIONS	71
	REFERENCES	73

1 INTRODUCTION

The increasing use of non-linear loads generates a flow of harmonics and interharmonics components in the electric power system. Their presence causes distortions in the currents and voltage waveforms, hence the power system becomes subject to many power quality disturbances. The harmonics components can be generated by many sources, including arc furnaces, inverters, rectifiers, and motor drivers, among others (IEEE, 2014). These components may have many adverse effects on the system, like the overheating of energy conductors and the reduction of the equipment lifetime (KUMAR et al., 2017). Therefore, to prevent the issues mentioned above, it is essential to develop algorithms for harmonics and interharmonics estimation even for maintenance of the system stability and efficiency on power delivery (SUN et al., 2020).

The use of a suitable algorithm for estimations is essential since its accuracy has a direct influence in the functioning of the system in several cases. The power systems protection has become increasingly digital, which enables protection devices to become more sensitive, faster and reliable (ORTBANDT et al., 2015). In order to contribute to the security and stability of the system, an accurate methodology must be applied to determine the system signals characteristics and allow the protection devices to work accurately and properly. An example of the importance of the algorithm accuracy is the blackout that occurred in Britain in 2019, where the frequency of the grid presented a deviation of more than 1% and caused an automatic shutdown, directly affecting million of people and severely disrupting the rail lines (AMBROSE, 2019). The detection algorithms worked properly and accurate, preventing larger damages to the system.

In addition, an accurate detection of the frequency components of the system may improve the power quality evaluation, as the identification of the source of disturbance (ZHAO et al., 2019) and has direct influence on the measured power consumed, which affects the value to be paid by the consumer (DUGAN et al., 2002). Also, accurate detection of the interharmonics in a system with distributed generation may provide a base for the impact and interference evaluation of these interharmonics emitted by a distributed system in the costumer voltage (RAVINDRAN et al., 2019). The current methodology adopted for measuring harmonics and interharmonics components in a power system is determined by IEC 61000-4-7

(IEC, 2006). A frequency resolution of 5 Hz is obtained by following the standard requirements, which may be a drawback for estimating interharmonics components using traditional Discrete Fourier Transform (DFT). This drawback occurs because of the *spectral leakage* phenomenon in the presence of interharmonics components and frequency deviations (TESTA et al., 2007).

In order to perform a robust and correct estimation several algorithms are proposed in the literature. Prony's method is commonly applied for harmonics and interharmonics estimation (ZYGARLICKI; MROCZKA, 2012; LOBOS et al., 2000; CHANG; CHEN, 2010). The Prony's method is similar to the Fourier Transform and decomposes the signal into a series of damped complex exponentials, but it is susceptible to noise present in the power system signals. To overcome this drawback, the Matrix Pencil Method (MPM), a variant of Prony's method first proposed by Hua e Sarkar (1988), can be employed to estimate the harmonics and interharmonics components (PRAMANIK; MITRA, 2017; SHESHYEKANI et al., 2017). It is showed that this algorithm is more robust to noise than the traditional Prony's method (RODRÍGUEZ et al., 2018) and recently Sheshyekani et al. (2017) and Pramanik e Mitra (2017) proposed methodologies based on MPM. Sheshyekani et al. (2017) proposed the solution of the generalized eigenvalue problem using MPM and Pramanik e Mitra (2017) proposed the Band-Pass Matrix Pencil Method, which consists of an IIR filtering method before the application of MPM for parameters estimation. The method proved to be able to estimate amplitude, phase and frequency without the grid-dependency. Nonetheless, it presents a high computational load and other methods already proved to be less noise-sensitive than MPM (PRADO et al., 2019).

Another widely used method is the estimation of signal parameters via the rotational invariance technique (ESPRIT) (IOZSA; VESA, 2010), a subspace method capable of achieving high-frequency resolution in order to characterize the signals parameters. However, as mentioned in Jin et al. (2018), the necessity of the model order is a remarkable drawback of this approach, since the model order is defined as the number of harmonics and interharmonics components present on the signal, so previous knowledge of the signal is required. A recently proposed methodology is the Fast Matching Pursuit (FMP) (LEI et al., 2017). The FMP is an iterative technique based on Sparse Signal Decomposition (SSD). As iterations occur, the components are identified based on the Fast Fourier Transform (FFT) spectrum of the signal until the algorithm achieves an acceptable value for the error. It proved to be an efficient methodology capable of extracting both harmonics and interharmonics from the power system signal even under fundamental frequency deviation conditions. Also, it has been demonstrated to avoid spectral leakage. However, it has been shown that the algorithm identifies non existent components in the power system signals under noisy conditions (PRADO et al., 2019).

The Harmonics and Interharmonics components Estimation based on Signal Sparse Decomposition (HIESSD) (PRADO et al., 2019) is another novel method based on SSD with an overcomplete dictionary. This algorithm can estimate the amplitude, phase, and frequency of harmonics and interharmonics components by applying the Sparse Signal Decomposition with the Orthogonal Matching Pursuit (OMP) algorithm. The algorithm proved to be robust under noise conditions and presented reduced error values. However, its behavior under frequency deviation is unpredictable since it is a fixed-grid based algorithm.

Because of the above, it is necessary to develop an algorithm that presents low error rate under noise and frequency deviation conditions without the previous knowledge of the model order and considering its execution time. Regarding this context, we propose a new algorithm for harmonics and interharmonics estimation in stationary signals called *Anti-leakage Matching Pursuit* (ALMP). The ALMP is an iterative algorithm based on the FMP algorithm. As the iterations happen, the signal's frequency components are extracted from the residue until a tolerance value is achieved, the selected components are chosen based on the establishment of a non-linear least squares problem that may be solved by an optimization technique. The ALMP algorithm overcome the DFT limitations concerning the interharmonics and frequency deviation estimations, being able to perform estimations under varied conditions as other algorithms well established in the literature, such as the MPM, but with a much lower computational load and complexity of implementation.

1.1 OBJECTIVES

The objective of this work is to develop an algorithm to correctly estimate harmonics and interharmonics in electric power systems even under conditions of fundamental frequency deviation and in the presence of white Gaussian noise.

The specific objectives of this works are:

- Literature review regarding the methods for harmonics and interharmonics estimation.
- Implementation of existing algorithms for harmonics and interharmonics estimation for a comparative study.
- Study and establishment of a non-linear least squares problem.
- Literature review regarding optimization algorithms.
- Comparative analysis between the optimization algorithms.

- Development of the novel method for estimating harmonic and interharmonics components: the Anti-Leakage Matching Pursuit (ALMP) algorithm.
- Research of real signals contaminated with harmonics, interharmonics and frequency deviation.
- Assessment of the ALMP results and comparison with the state-of-the-art methods using power system signals.

1.2 OUTLINE OF THE DOCUMENT

This work is divided in five chapters. Chapter 2 presents the literature review, which includes an introduction of harmonics, interharmonics and frequency deviation sources and causes, the theoretical foundation necessary to the good comprehension of the proposed methodology consisting of a theoretical basis of optimization algorithms and non-linear least squares problems. Chapter 3 presents the methods used for harmonics and interharmonics estimation that constitute the state-of-the-art. Chapter 4 presents the proposed ALMP algorithm and each step performed by the algorithm in details. Chapter 5 presents the results obtained with ALMP algorithm and the results from other state-of-the-art algorithms for the same signals in order to establish a comparison. Finally, the final considerations are presented in Chapter 5.

2 THEORETICAL FOUNDATION

This chapter presents the theoretical basis of the work and the topics covered here are based on the main steps of the proposed algorithm. Figure 1 presents a simplified flowchart representing the ALMP algorithm:

Figure 1: Simplified flowchart representing ALMP's steps.

First, a theoretical base for harmonics and interharmonics distortions is presented, as well as their sources, since the algorithm is first initialized to process a signal contaminated with harmonics, interharmonics, noise and/or frequency deviation. Then, a review of the main approaches used for unconstrained optimization problems takes place, since it will be necessary for the comprehension of the main stage of the developed algorithm ALMP: the *Parameters Estimation*. This step is based on the solution of a non-linear least squares problem that may be solved by an optimization algorithm for unconstrained problems, more details about ALMP will be presented in Chapter 4.

2.1 HARMONICS AND INTERHARMONICS DISTORTIONS

The increase of the use of non-linear loads connected to electric power systems modifies the alternating current (AC), adding harmonic currents on the power system. These currents can cause damage to equipment and also interference in many communication equip-

ment. Therefore, power systems are subject to many power quality disturbances, which can cause distortions in the waveforms present in the system that can be analyzed by the spectral content of the signal (IEEE, 2014).

Harmonics components of frequency are defined as integer multiples of the fundamental frequency, and interharmonics components are non-integer multiples of the fundamental frequency (IEEE, 2014). Many sources can generate harmonics and interharmonics distortions, and some of those sources are described in the subsection below.

2.1.1 Harmonics Distortions Sources

The use of non-linear loads generates a flow of harmonics currents in the electric power system. According to IEEE (2014), the sources of harmonics currents include: arc furnaces, static VAR systems, inverters for distributed generation, AC phase controllers, cyclo converters, and AC-DC converters (rectifiers) commonly used in switched-mode power supplies and pulse width modulated motor drivers. The following presents three harmonic sources that will be used to evaluate this work in greater detail: overexcited transformer, single-phase dc source and six-pulse converter.

2.1.1.1 Overexcited transformer

Usually, in electrical distribution systems, the power transformer may operate under a voltage level above its nominal, which generates harmonic distortion caused by the increase in the magnetizing current. Arrillaga e Watson (2003) presented the conditions used for modeling the magnetizing current of the transformer under 30% of overexcitation. The generated signal $y(t)$ is presented in Eq. (1)

$$y(t) = \sin(\omega t) - 0.18 \sin(3\omega t) + 0.11 \sin(5\omega t), \quad (1)$$

where the angular frequency ω is equal to $2\pi f$, and f the fundamental frequency considered here equal to 60 Hz. Figure 2 shows an overexcited transformer's magnetizing current signal measured in the primary winding and its spectrum in frequency obtained with the DFT algorithm performed according to IEC 61000-4-7 (IEC, 2006) with a 200 ms sampling window, sampling rate of 256 samples per cycle which results in a sampling frequency F_s of 15360 Hz and number of samples $N = 3072$.

Figure 2: The overexcited transformer's magnetizing current signal.

2.1.1.2 Single-phase direct current source

A current signal from a single-phase direct current (DC) source is considered and presented in Eq. (2), which models the supply current I_h from the source by an uncontrolled single-phase rectifier (BOLLEN; GU, 2006), i.e., a diode rectifier, whose input consists of an alternating current (AC) that is converted into DC.

$$i_h = \begin{cases} +\frac{2\sqrt{2}}{h}i_{DC} & h = 1, 5, 9, 13, \dots \\ -\frac{2\sqrt{2}}{h}i_{DC} & h = 3, 7, 11, 15, \dots \\ 0 & h = 0, 2, 4, 6, \dots \end{cases} \quad (2)$$

where i_{DC} is the maximum value of the charge current and h indicates the harmonic component. Figure 3 shows the signal and its spectrum in frequency obtained with DFT algorithm performed according to IEC 61000-4-7 (IEC, 2006) with a 200 ms sampling window, sampling rate of 256 samples per cycle, sampling frequency of 15360 Hz and $N = 3072$. This signal only has frequency components multiple of 5 Hz, which enables the correct estimation with DFT methods without considering the presence of noise.

Figure 3: The single-phase DC source's input current signal.

2.1.1.3 Six-pulse converter

The AC/DC converters are widely used in industrial applications as the power signals rectification. The Eq. (3) presents the harmonic current components i_h generated by a six pulse rectifier with a 10 degrees trigger angle (RICE, 1994).

$$i_h = \frac{1}{h} i_1, \quad \text{for } h = 6j \pm 1 \text{ and } j = 1, 2, 3, \dots, \quad (3)$$

where i_1 is the fundamental current. Figure 4 shows the current signal and its spectrum in frequency obtained with the DFT algorithm performed according to IEC 61000-4-7 (IEC, 2006) with a 200 ms sampling window, sampling rate of 256 samples per cycle, sampling frequency of 15360 Hz and $N = 3072$.

Figure 4: The six-pulse converter's current signal.

2.1.2 Interharmonics Distortions Sources

Interharmonics currents can appear in an electrical system from many sources. They are formed mainly by equipment that causes random changes in the current or voltage. That is, equipment operating in transient state or current/voltage modulation systems for control purposes. According to Marz (2016), some examples of interharmonics sources include arcing loads, induction motors, electronic frequency converters, variable load drivers, and voltage source converters. Below are two additional examples presented in greater detail; the cycloconverter and the synchronous machine.

2.1.2.1 Cycloconverter

Cycloconverters are usually used to feed high-power synchronous motor drivers. The interharmonic frequencies generated by them depends on the speed of the motor and are related to the desired frequency at the motor side (SYAM et al., 2004). Syam et al. (2004) presented an experimental study of the input supply current of a cycloconverter, where the authors consider the fundamental frequency to be 50 Hz with a deviation of 0.75 Hz and the desired output of the cycloconverter of 35 Hz. Considering these conditions, the input supply

signal of a synchronous motor fed by a circulating current-free type cycloconverter obtained experimentally may be represented by Eq. (4).

$$y(t) = 0.66 \sin(2\pi f_1 t) + 0.48 \sin(2\pi(2f_1 - 3f_O)t) + 0.31 \sin(2\pi(4f_1 - 3f_O)t), \quad (4)$$

where f_O is the output frequency of the cycloconverter and f_1 is the fundamental frequency. Figure 5 shows the signal and its spectrum in frequency obtained with the DFT algorithm performed according to IEC 61000-4-7 (IEC, 2006) with a 200 ms sampling window, sampling rate of 256 samples per cycle, sampling frequency of 15360 Hz and $N = 3072$. It can be observed that the DFT algorithm could not correctly identify all the harmonics and interharmonics components due to a phenomena called *Spectral Leakage*, where there is the energy scattering of the estimated component to the adjacent components.

Figure 5: The Cycloconverter's output signal.

2.1.2.2 Synchronous Machine

Synchronous machines present a non-sinusoidal variation of the mutual inductance between the rotor and stator, also unbalance in the rotor phase winding, these factors may generate interharmonic currents on the system. Also, the harmonics and interharmonics identification from electric machines is essential, since the frequencies present in the spectrum from the machine's current may indicate the presence and severity of a failure in the motor (SPYROPOULOS; MITRONIKAS, 2013). Wallace et al. (1974) established the stator current signal is composed of the fundamental component in 60 Hz, two subharmonic components at 24 and 48 Hz, and five interharmonic components at 96, 264, 384, 588, and 708 Hz, as presented in Eq. (5),

$$\begin{aligned} i_s(t) = & 0.03 \sin(2\pi 24t) + 0.024 \sin(2\pi 48t) + \sin(2\pi 60t) \\ & + 0.023 \sin(2\pi 96t) + 0.029 \sin(2\pi 264t) + 0.03 \sin(2\pi 384t) \\ & + 0.003 \sin(2\pi 588t) + 0.004 \sin(2\pi 708t). \end{aligned} \quad (5)$$

Figure 6 shows the signal and its spectrum in frequency obtained with the DFT algorithm performed according to IEC 61000-4-7 (IEC, 2006) with a 200 ms sampling window,

sampling rate of 256 samples per cycle, sampling frequency of 15360 Hz and $N = 3072$. The *Spectral Leakage* can be observed, due to the fixed resolution of the DFT algorithm.

Figure 6: The Synchronous Machine's stator signal.

2.2 FREQUENCY DEVIATION ON POWER SYSTEMS

The increasing use of renewable energy sources may cause fluctuations in power grids, especially the two main ones: wind and solar. The power generation from a wind farm, for example, presents variations and may not be able to supply the energy demand. In this case the kinetic energy of the generator is converted into electric energy and the frequency of the system decreases (SCHÄFER et al., 2018). Besides an imbalance between power generation and consumption, the frequency deviation may be caused by fluctuating demand and energy trading (SCHÄFER et al., 2018).

Therefore, the use of distributed energy resources as occurs in microgrids enables them to be a generator of frequency deviations, at the moment of connection of the microgrid to the network (WANG et al., 2012) and in island mode operation due to the power variation of the generation (SCHÄFER et al., 2018). The microgrids present nonlinear loads and power electronics devices connected to it, which generate harmonics and interharmonics signals. Also, it is susceptible to fundamental frequency deviation, due to the power control strategies applied in the distributed generation at the moment the grid connects to the network. Wang et al. (2012) presented the simulation of a power grid system with photovoltaic generation. The signal modeled by Wang et al. (2012) considers a deviation of 0.2 Hz in the fundamental frequency of 50 Hz, and the presence of two harmonics of the order 5 and 7 and three interharmonics of order 0.4, 4.6 and 6.8. The normalized current signal from the point of common connection considering the components with relevant amplitude can be modeled

according to Eq. (6),

$$\begin{aligned}
 y(t) = & 0.0026 \cos(2\pi 19.92t - 157.34^\circ) + \cos(2\pi 49.8t + 147.49^\circ) \\
 & + 0.0015 \cos(2\pi 229.08t + 78.94^\circ) + 0.0302 \cos(2\pi 249t + 101.91^\circ) \\
 & + 0.0245 \cos(2\pi 348.6t - 145.68^\circ).
 \end{aligned} \tag{6}$$

The signal and its spectrum in frequency obtained with the DFT algorithm is showed in Figure 7, the spectrum was obtained considering the IEC 61000-4-7 (IEC, 2006) with a 200 ms sampling window, sampling rate of 256 samples per cycle, sampling frequency of 15360 Hz and $N = 3072$. The *spectral leakage* is observed, since the resolution of the DFT algorithm is fixed in 5 Hz.

(a) In time-domain.

(b) In frequency-domain, not presenting the fundamental frequency of 49.8 Hz.

Figure 7: Signal from a microgrid with fundamental frequency deviation, harmonics and interharmonics.

2.3 UNCONSTRAINED OPTIMIZATION

Optimization algorithms are widely used to maximize or minimize a determined value of a function, and this function is called the *objective function*. The objective function relies on *system variables*, which are modified by an *optimization algorithm* in order to optimize the value assumed by the objective function. The optimization problems can be classified into one of two categories: constrained and unconstrained problems. These definitions refer to the feasibility of the variable to assume any value in $\mathbb{R}^n$. In unconstrained problems, the variables may assume any value. In contrast, the constrained problems, there are restrictions; for example, an optimization problem formulated in order to minimize the time to perform a task can not assume negative values. The ALMP deals only with unconstrained problems, so the constrained optimization problems will not be addressed.

The mathematical formulation of the unconstrained optimization is given in Eq. (7)

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} F(\mathbf{x}), \tag{7}$$

where $\mathbf{x} \in \mathbb{R}^N$ is the variables vector, $F : \mathbb{R}^N \rightarrow \mathbb{R}$ corresponds to the objective function, and $\hat{\mathbf{x}} \in \mathbb{R}^N$ is the optimization problem solution. Most optimization algorithms are iterative and initiate from a given start point $\mathbf{x}_0$. The algorithms use the information about the objective function F , and the values of early iterations $\mathbf{x}_0, \mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_{k-1}$ to determine $\mathbf{x}_k$, that corresponds to the problem's solution at iteration k . There are two main approaches to determine the direction of the next iteration value ($\mathbf{x}_k \rightarrow \mathbf{x}_{k+1}$): line search and trust-region.

2.3.1 Line Search Methods

In the line search approach, the algorithm chooses a direction $\mathbf{p}_k \in \mathbb{R}^N$ and aims to obtain an optimal step length $\alpha \in \mathbb{R}$ that minimizes the objective function by solving the problem presented in Eq. (8) (NOCEDAL; WRIGHT, 2006):

$$\hat{\alpha} = \arg \min_{\alpha > 0} F(\mathbf{x}_k + \alpha \mathbf{p}_k), \quad (8)$$

where $\mathbf{x}_k$ refers to the current iterate. Although the solution of Eq. (8) represents the ideal step value, the search would generate a large cost, so the line search algorithm limits the number of step values and among the trial ones it selects the step that best minimizes Eq. (7). One way of determine the step direction $\mathbf{p}_k$ is applying the *Newton direction*, which can be approximated by the second-order Taylor series:

$$m_k(\mathbf{p}) = F(\mathbf{x}_k + \mathbf{p}) \approx F_k + \mathbf{p}^T \nabla F_k + \frac{1}{2} \mathbf{p}^T \nabla^2 F_k \mathbf{p}, \quad (9)$$

where $m_k(\cdot)$ is the quadratic model, F_k is a scalar equivalent to the value of the objective function at the point $\mathbf{x}_k$, which refers to the value of the current solution, ∇F_k is the gradient vector of the objective function at the point $\mathbf{x}_k$, $\nabla^2 F_k$ is the Hessian matrix of the objective function at the point $\mathbf{x}_k$, $(\cdot)^T$ is the transpose operation and $\mathbf{p}$ is the vector that minimizes $m_k(\mathbf{p})$. The Newton's direction assumes the Hessian matrix being positive definite¹ and the direction is obtained by finding the vector $\mathbf{p}$ that minimizes Eq. (9), which may be computed by equaling to zero the derivative of the expression presented in Eq. (9). Hence, the minimum is obtained by Eq. (10),

$$\mathbf{p}_k = -(\nabla^2 F_k)^{-1} \nabla F_k. \quad (10)$$

This approximation presents high accuracy when $\|\mathbf{p}\|$ is small and $\nabla^2 F_k$ is positive definite (NOCEDAL; WRIGHT, 2006). Nonetheless, the Hessian calculation may be an expensive process, which leads to the use of alternatives methods for searching directions, as the *Quasi-Newton* (QN) methods. The QN methods do not need the Hessian, since they use an

¹A positive definite matrix is symmetric and all its eigenvalues are positive (AYRES, 1962).

approximation B_k , that considers the approximation of the Hessian as presented in Eq. (11):

$$\nabla^2 F_k(\mathbf{x}_{k+1} - \mathbf{x}_k) \approx \nabla F_{k+1} - \nabla F_k. \quad (11)$$

The B_k is approximated by the *secant equation*, as presented in Eq. (12):

$$B_{k+1} \mathbf{s}_k = \nabla F_{k+1} - \nabla F_k, \quad (12)$$

where $\mathbf{s}_k$ refers to $\mathbf{x}_{k+1} - \mathbf{x}_k$. There are several methods that can be used to approximate the Hessian, some examples are the Symmetric Rank-One (SR1) formula (CONN et al., 1991), Berndt–Hall–Hall–Hausman (BHHH) (CAUDILL, 2006) method and Broyden–Fletcher–Goldfarb–Shanno (BFGS) method (NOCEDAL; WRIGHT, 2006), of which BFGS is the most used QN method and will be explained in more detail in Section 2.4.3.

2.3.2 Trust-Region Methods

The trust-region methods consider a model $m_k(\cdot)$ that presents a behavior similar to the objective function near the current point $\mathbf{x}_k$, thus the search for a minimum is restricted to the region near $\mathbf{x}_k$. The trust-region minimization problem is presented in Eq. (13):

$$\hat{\mathbf{p}} = \arg \min_{\mathbf{p}} m_k(\mathbf{x}_k + \mathbf{p}), \quad (13)$$

where $\mathbf{x}_k + \mathbf{p}$ is considered under a *trust region* in order to maintain $m_k(\cdot)$ near $F(\cdot)$. The $\hat{\mathbf{p}}$ obtained by Eq. (13) represents the solution for the sub-problem, and if it does not decrease $F(\cdot)$ as desired, the trust region is decreased, and the process is repeated. Usually, the $m_k(\cdot)$ is defined analogously to that presented in Eq. (9), Eq. (14) shows $m_k(\cdot)$:

$$m_k(\mathbf{x}_k + \mathbf{p}) = F_k + \mathbf{p}^T \nabla F_k + \frac{1}{2} \mathbf{p}^T B_k \mathbf{p}, \quad (14)$$

where B_k may be the Hessian or an approximation of it. Differently from the presented line search approach, the direction may change at each iteration in the trust-region method. First, the method defines a step-length and determines the best direction for it. The *trust region* is usually considered a ball defined by the expression shown in Eq. (15) (NOCEDAL; WRIGHT, 2006):

$$\|\mathbf{p}\|_2 \leq \Delta, \quad (15)$$

where $\Delta > 0$ is the trust-region radius.

Figure 8 shows the steps performed by two optimization algorithms based on trust region and line search approaches to find the optimum solution that minimizes the objective

function $F(\cdot)$, the contours of $F(\cdot)$ are represented by the continuous lines. The model $m_k(\cdot)$ has its contours represented by the dashed elliptical forms. The line search approach aims to find along the step the $m_k(\cdot)$ minimizer, while the trust region approach minimized the $m_k(\cdot)$ inside the trust region. In this case, the trust region approach, due to its characteristics, led to a reduction of $F(\cdot)$ larger than the line search approach.

Figure 8: Steps of unconstrained optimization algorithms based on trust-region and line search methods for $x_k \in \mathbb{R}^2$. Adapted from (NOCEDAL; WRIGHT, 2006).

2.4 NON-LINEAR LEAST SQUARES PROBLEMS

The objective function may assume a special form called non-linear least-squares problem (GRIVA et al., 1996). The non-linear least squares problem assume the form of an unconstrained optimization problem that aims to minimize the sum of squares of the residue, i.e., the difference between the measured signal $\mathbf{y}$ and a non-linear mathematical model $g(\mathbf{x})$, as represented by Eq. (16),

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \|\mathbf{y} - g(\mathbf{x})\|^2. \quad (16)$$

The non-linear problems are based on the differentiable and non-linear residue of the data. As explained in more detail in the next Chapter, the ALMP algorithm establishes a non-linear least square problem and optimization algorithms suitable for solving this kind of problems must be analyzed. Three main optimization algorithms present in literature are considered: the Levenberg-Marquadt, the Nelder-Mead and the Quasi-Newton Algorithm. These algorithms were chosen to deeper studies since they constitute the state-of-the-art of the optimization algorithms for unconstrained optimization problems, being the Levenberg-Marquadt a trust-region approach, the Quasi-Newton, a linear search approach and the Nelder-Mead a derivative-free approach and already proposed by the FMP algorithm.

2.4.1 Levenberg-Marquadt Algorithm

The Levenberg-Marquadt algorithm (LMA) (LOURAKIS, 2005) is a widely used optimization technique (CHRYSOCHOS et al., 2014; YANG; ZOU, 2020; QIAO et al., 2018) and already proved to provide satisfactory estimates (ALAMO, 1993) and to be numerically advantageous (CHRYSOCHOS et al., 2014). It is a trust-region technique that combines the advantages of the Gauss-Newton method and steepest descent method for the solution of non-linear least-squares problems.

Given the objective function $F(\cdot)$ that relates the estimated vector $\hat{x}$ with the parameter vector θ , based on the vector of initial parameters (θ_0) and in the measured vector x , it is desired to minimize the squared distance presented in Eq. (17)

$$\mathbf{r}^T \mathbf{r}, \quad (17)$$

where $\mathbf{r} = \mathbf{x} - \hat{\mathbf{x}}$. The LMA performs a linear approximation in terms of θ considering the value δ_θ . The Eq. (18) presents the Taylor series expansion for the approximation until the first-order term:

$$F(\theta + \delta_\theta) \approx F(\theta) + \mathbf{J} \delta_\theta, \quad (18)$$

where $\mathbf{J}$ is the Jacobian matrix $\frac{\partial F(\theta)}{\partial \theta}$. The LMA is an iterative method that aims to converge towards a local minimum θ^+ by finding the ideal δ_θ that minimizes Eq. (19),

$$\|\mathbf{r} - \mathbf{J} \delta_\theta\|_2, \quad (19)$$

where, δ_θ can be found as the solution of the normal equations presented in Eq. (20):

$$\mathbf{J}^T \mathbf{J} \delta_\theta = \mathbf{J}^T \mathbf{r}. \quad (20)$$

However, the LMA aims to find the solution of a variation of Eq. (20) called *augmented normal equations*, where the Hessian is replaced by $(\mathbf{J}^T \mathbf{J} + \lambda \mathbf{I})$, as presented in Eq. (21). The damping term λ is introduced, since it allows adjustments at each iteration of the LMA

$$(\mathbf{J}^T \mathbf{J} + \lambda) \delta_\theta = \mathbf{J}^T \mathbf{r}. \quad (21)$$

The Eq. (21) is computed, if the parameter vector obtained with the δ_θ obtained reduces the residue $\mathbf{r}$, the damping factor can be reduced and the process is repeated. Otherwise the damping factor is increased until the algorithm find a δ_θ that decreases the error. A more detailed description is presented in (LOURAKIS, 2005) and (RANGANATHAN, 2004).

The main disadvantage of the LMA concerns the initial value, if it is far from the optimum solution, the algorithm may present a high time for convergence (LOURAKIS, 2005). However, a first estimation is performed in order to obtain the initial point through the Fourier Transform and the optimum value can be found in a ratio 5 Hz, considering the IEC 61000-4-7, which guarantees an initial point close to the solution.

2.4.2 Nelder-Mead Algorithm

The Nelder-Mead Algorithm (NMA) (SINGER; NELDER, 2009) is a derivative-free method applied for finding a local minimum of a function with T variables. NMA is a technique widely used (LEI et al., 2017; CHEN et al., 2018; BAUDZUS; KRUMMRICH, 2019) due to its efficiency and computational compactness (MATHEWS; FINK, 2004). In this method, the minimum point is searched through several steps performed by the algorithm: *reflection*, *expansion*, *contraction* and *shrinkage* (SINGER; NELDER, 2009).

The NMA is a simplex-based method (NELDER; MEAD, 1965), i.e., the variables can be considered the vertices of a polygon called *simplex*, Figure 9 illustrates this principle. The first step of NMA is the construction of the simplex based on the given point x_0 .

Figure 9: Polygons considered by Nelder-Mead algorithm in $\mathbb{R}^2$ and $\mathbb{R}^3$.

Next, the Simplex Transformation Algorithm is applied, this algorithm is based in three main steps. First the vertices index are ordered from the best to the worst case. The best vertex minimizes the objective function $F(\cdot)$, while the worst vertex maximize it. The second main step consists of the computation of the best side's centroid. Finally, the last main step of transformation is performed. In the last step a new simplex is computed based on the current one, at maximum four operations are performed in this step in order to establish a better point for the worst vertex as follows:

- *Reflection*;
- *Expansion*;

- *Contraction*;
- *Shrinkage*.

Given a triangle formed by three points: the best case (x_b), the second best case (x_s) and the worst case (x_w). Typically, x_0 is equal to the initial point, and the other points are calculated based on an arbitrary step value. The best case point is the one that results in the minimum objective function value, the worst point corresponds to the maximum function value and the second best case corresponds to the intermediate function value considering the three cases.

The *Reflection* operation presented in Figure 10(a) computes a reflected point x_r based on a parameter $\alpha > 0$ and if $F(x_b) \leq F(x_r) < F(x_s)$, the reflected point x_r is accepted and the iterations are terminated. Otherwise, if $F(x_r) < F(x_b)$, the *Expansion* is performed as showed in Figure 10(b). In this operation an expanded point x_e is obtained based on the x_r and on a parameter $\gamma > 1$. If $F(x_e) < F(x_r)$ then the expanded point x_e is accepted, else the reflected point x_r is accepted and the operation terminates.

If $F(x_r) \geq F(x_s)$ then the *Contraction* takes place, Figure 10(c), and compute a contracted point x_c based on the reflected point x_r or the worst case point x_w and on a contraction term $0 < \beta < 1$. If $F(x_s) \leq F(x_r) < F(x_w)$ and $F(x_c) \leq F(x_r)$, then x_c is accepted and the iterations terminated, else the *Shrinkage* operation takes place.

If $F(x_r) \geq F(x_w)$ and $F(x_c) \leq F(x_w)$, then x_c is accepted. Otherwise, the *Shrinkage* operation is performed, as shown in Figure 10(d). In the *Shrinkage* operation two new vertices are computed based on a $0 < \delta < 1$ term. Figure 10 illustrates the four operations.

Figure 10: Operations performed in the Transformation step: (a) Reflection; (b) Expansion; (c) Contraction; (d) Shrinkage.

Despite its simplicity for implementation and understanding, the NMA presents limitations concerning the convergence. McKinnon (1996) proved through functions examples that the method does not present a global convergence.

2.4.3 Quasi-Newton Method

A widely used method (LI; LIU, 2019; ZHENG; WU, 2019; SALMAN et al., 2020) for non-linear unconstrained problems is the already presented Quasi-Newton (QN) Method (NOCEDAL; WRIGHT, 2006). Here is presented the most popular QN algorithm: the BFGS method, developed by Broyden, Fletcher, Goldfarb and Shanno. Recalling the quadratic model presented in Eq. (14)

$$m_k(\mathbf{p}) = F_k + \nabla F_k^T \mathbf{p} + \frac{1}{2} \mathbf{p}^T \mathbf{B}_k \mathbf{p},$$

it is possible to state the minimizer $\mathbf{p}_k$ of this function, which is shown in Eq. (22).

$$\mathbf{p}_k = -\mathbf{B}_k^{-1} \nabla F_k, \quad (22)$$

this minimizer is used to calculate the new iterate, which makes this method very similar to the Newton's method, differing in terms of $\mathbf{B}_k$. Eq. (23) express the step performed at each iteration based on the minimizer,

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \alpha_k \mathbf{p}_k. \quad (23)$$

In order to compute $\mathbf{B}_k$ without calculating it at each iteration, first it is considered a new model for the iteration $\mathbf{x}_{k+1}$, as presented in Eq. (24)

$$m_{k+1}(\mathbf{p}) = F_{k+1} + \nabla F_{k+1}^T \mathbf{p} + \frac{1}{2} \mathbf{p}^T \mathbf{B}_{k+1} \mathbf{p}. \quad (24)$$

It can be seen that $\nabla m_{k+1}(\mathbf{0}) = \nabla f_{k+1}$, hence, considering Eq. (23), it is possible to obtain the expression presented in Eq. (25).

$$\nabla m_{k+1}(-\alpha_k \mathbf{p}_k) = \nabla F_{k+1} - \alpha_k \mathbf{B}_{k+1} \mathbf{p}_k = \nabla F_k. \quad (25)$$

By rearranging the Eq. (25), it is possible to obtain the already presented *secant equation*:

$$\mathbf{B}_{k+1} \alpha_k \mathbf{p}_k = \nabla F_{k+1} - \nabla F_k \rightarrow \mathbf{B}_{k+1} \mathbf{s}_k = \nabla F_{k+1} - \nabla F_k,$$

where $\mathbf{s}_k = \alpha_k \mathbf{p}_k$ and $\mathbf{y}_k = \nabla f_{k+1} - \nabla f_k$. The BFGS method considers the secant equation defined in terms of the Hessian:

$$\mathbf{H}_{k+1}(\nabla F_{k+1} - \nabla F_k) = \mathbf{s}_k. \quad (26)$$

The BFGS method considers that the following condition must be satisfied:

$$\min_{\mathbf{H}} \|\mathbf{H} - \mathbf{H}_k\| \quad \text{subject to } \mathbf{H} = \mathbf{H}^T, \quad \mathbf{H} \mathbf{y}_k = \mathbf{s}_k. \quad (27)$$

The solution for Eq. (27) is the BFGS formula and is given by:

$$\mathbf{H}_{k+1} = (\mathbf{I} - \rho_k \mathbf{s}_k \mathbf{y}_k^T) \mathbf{H}_k (\mathbf{I} - \rho_k \mathbf{y}_k \mathbf{s}_k^T) + \rho_k \mathbf{s}_k \mathbf{s}_k^T, \quad (28)$$

where $\rho_k = (\mathbf{y}_k^T \mathbf{s}_k)^{-1}$. The Eq. (28) can be rewrite using the Sherman–Morrison–Woodbury formula (NOCEDAL; WRIGHT, 2006), that states a matrix $\hat{\mathbf{A}}$ represented by Eq. (29),

$$\hat{\mathbf{A}} = \mathbf{A} + \mathbf{U} \mathbf{V}^T, \quad (29)$$

where $\mathbf{U}, \mathbf{V} \in \mathbb{R}^{N \times P}$, being P between 1 and N , $\mathbf{A}$ is a non-singular square matrix and $\hat{\mathbf{A}}$ is a square matrix with non-singularity defined by the non-singularity of $(\mathbf{I} + \mathbf{V}^T \mathbf{A}^{-1} \mathbf{U})$. The Sherman–Morrison–Woodbury formula defines the inverse of $\hat{\mathbf{A}}$ can be expressed as presented in Eq. (30)

$$\hat{\mathbf{A}}^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1} \mathbf{U} (\mathbf{I} + \mathbf{V}^T \mathbf{A}^{-1} \mathbf{U})^{-1} \mathbf{V}^T \mathbf{A}^{-1}. \quad (30)$$

Applying (30) in (28) it is possible to derive a BFGS formula that works with the approximation $\mathbf{B}_k$, Eq. (31) presents the BFGS formula

$$\mathbf{B}_{k+1} = \mathbf{B}_k - \frac{\mathbf{B}_k \mathbf{s}_k \mathbf{s}_k^T \mathbf{B}_k}{\mathbf{s}_k^T \mathbf{B}_k \mathbf{s}_k} + \frac{\mathbf{y}_k \mathbf{y}_k^T}{\mathbf{y}_k^T \mathbf{s}_k}. \quad (31)$$

BFGS is the most popular Quasi-Newton method, but it presents a major drawback concerning the initial point. If the initial point is far from the solution, the iterative matrices used by the method may not be positives definite, which hinders the global convergence of the method (ZHOU; ZHANG, 2010). However, as explained for LMA algorithm, this may not affect the proposed algorithm, since the initial point is settled under the resolution established by IEC 61000-4-7 (IEC, 2006).

2.5 CHAPTER REVIEW

This chapter presents the theoretical basis of harmonics and interharmonics distortions and some examples of sources of wave distortions: the overexcited transformer, the single-phase DC source, the six pulse converter, the cycloconverter, and the synchronous machine, as well as a source of frequency deviation and interharmonics, the microgrid systems. Also, the theoretical basis of unconstrained optimization algorithms was presented to establish concepts used in the proposed algorithm.

3 METHODS FOR ESTIMATING FREQUENCY COMPONENTS

The most used and recent methods for harmonics and interharmonics estimation are presented here in more detail. Those methods are the Discrete Fourier Transform, Matrix Pencil Method, Matching Pursuit, Harmonics and Interharmonics components Estimation based on Signal Sparse Decomposition and the Fast Matching Pursuit.

3.1 DISCRETE FOURIER TRANSFORM

The most common method used to estimate the frequency components of a signal is the DFT, which is an efficient and convenient tool for fast computation of convolutions (OPPENHEIM, 1999). The definition of the DFT is represented in Eq. (32),

$$\tilde{Y}[k] = \sum_{n=0}^{N-1} y[n] W_N^{kn} \quad k \in 0, 1, \dots, N-1. \quad (32)$$

where $W_N = e^{-2\pi j/N}$, N represents the period of the signal and j the imaginary unit, $\tilde{Y}[k]$ the Fourier coefficients and $y[n]$ corresponds to a periodic sequence.

Considering IEC 61000-4-7 (IEC, 2006), only frequencies multiple of 5 Hz would be correctly identified by the DFT, causing the spectrum leakage for components non-multiple of 5 Hz. Figure 11 shows this phenomenon for a sinusoidal signal with frequency deviation of 2.5 Hz ($f = 62.5$ Hz) and a unit amplitude with a sampling window of 200 ms. Spectral leakage makes it challenging to identify the components of the signal, and it is not possible to identify components that are non-multiple of the fundamental frequency.

Figure 11: Spectral Leakage in DFT algorithm for a frequency out of the frequency grid.

3.2 MATRIX PENCIL METHOD

The MPM is a technique used for estimating harmonics and interharmonics in power systems (SHESHYEKANI et al., 2017). It has been proved that it possesses advantages over its predecessor, the Prony's method, such as its better computational efficiency and lower sensitivity to noise (RODRÍGUEZ et al., 2018). The MPM method is based on the solution of an eigenvalue problem and it has been proved to provide high-resolution solutions for estimating harmonics and interharmonics (SHESHYEKANI et al., 2017). A power system signal in time-domain $y(t)$ without presence of noise is considered, as presented in Eq. (33),

$$y(t) \approx \sum_{i=1}^M R_i e^{s_i t}, \quad (33)$$

where, $R_i = -a_i + jb_i$ and $s_i = -\zeta_i + j\omega_i$, ζ_i being the damping factor and ω_i being the angular frequency, and M the model order. The signal is sampled with a sampling time T_s and can be written as

$$y[kT_s] \approx \sum_{i=1}^M R_i z_i^k, \quad (34)$$

for $k = 0, 1, \dots, N - 1$ and $z_i = e^{(\zeta_i + j\omega_i)T_s}$.

Eq. (34) represents the problem which MPM pursue to solve. In order to resolve the issue, the method proposes the construction of the two matrices showed in Eq. (35) and (36),

$$\mathbf{Y}_1 = \begin{bmatrix} y_1 & y_2 & \dots & y_L \\ y_2 & y_3 & \dots & y_{L+1} \\ \vdots & \vdots & \dots & \vdots \\ y_{N-L} & y_{N-L+1} & \dots & y_{N-1} \end{bmatrix}, \quad (35)$$

$$\mathbf{Y}_2 = \begin{bmatrix} y_0 & y_1 & \dots & y_{L-1} \\ y_1 & y_2 & \dots & y_L \\ \vdots & \vdots & \dots & \vdots \\ y_{N-L-1} & y_{N-L} & \dots & y_{N-2} \end{bmatrix}. \quad (36)$$

The pencil parameter is represented by L , chosen arbitrarily. Furthermore, the presented matrices can also be written as Eq. (37) and (38),

$$\mathbf{Y}_{1(N-L) \times L} = \mathbf{Z}_{1(N-L) \times M} \mathbf{R}_{M \times M} \mathbf{Z}_{2M \times L}, \quad (37)$$

$$\mathbf{Y}_{2(N-L) \times L} = \mathbf{Z}_{1(N-L) \times M} \mathbf{R}_{M \times M} \mathbf{Z}_{0M \times M} \mathbf{Z}_{2M \times L}, \quad (38)$$

where $\mathbf{Z}_1$, $\mathbf{Z}_2$ and $\mathbf{Z}_0$ are based in z_i presented in Eq. (34) and $\mathbf{R}$ is based in R_i of the same equation. The referred parameters are represented by the following matrices:

$$\mathbf{Z}_1 = \begin{bmatrix} 1 & 1 & \dots & 1 \\ z_1 & z_2 & \dots & z_M \\ \vdots & \vdots & \dots & \vdots \\ z_1^{N-L-1} & z_2^{N-L-1} & \dots & z_M^{N-L-1} \end{bmatrix}, \quad (39)$$

$$\mathbf{Z}_2 = \begin{bmatrix} 1 & z_1 & \dots & z_1^{L-1} \\ 1 & z_2 & \dots & z_2^{L-1} \\ \vdots & \vdots & \dots & \vdots \\ 1 & z_M & \dots & z_M^{L-1} \end{bmatrix}, \quad (40)$$

$$\mathbf{Z}_0 = \text{diag}(z_1, z_2, \dots, z_M), \quad (41)$$

$$\mathbf{R} = \text{diag}(R_1, R_2, \dots, R_M), \quad (42)$$

where $\text{diag}(\cdot)$ corresponds to a diagonal matrix. The MPM is then formulated as presented in Eq. (43):

$$\mathbf{Y}_2 - \lambda \mathbf{Y}_1 = \mathbf{0}, \quad (43)$$

where λ represents the eigenvalues and are equivalents to the z_i parameters. When $\lambda = z_i$, the matrix becomes singular, since its rank is $M - 1$ due to i th equal to zero. In this case, Ikramov (1993) showed that is possible to consider the solution equivalent to the eigenvalues of the pair $\{\mathbf{Y}_2, \mathbf{Y}_1\}$. Otherwise, the solution is given by the ordinary eigenvalue problem $\mathbf{Y}_1^+ \mathbf{Y}_2 - \lambda \mathbf{I}$, where $[\cdot]^+$ is the Moore-Penrose pseudo-inverse operation (IKRAMOV, 1993). Therefore, the damping factor and frequency of the components can be found, as well as the complex amplitudes. Hence, the amplitudes and phases can be obtained by solving the Least Square problem of Eq. (44)

$$\begin{bmatrix} y_0 \\ y_1 \\ \vdots \\ y_{N-1} \end{bmatrix} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ z_1 & z_2 & \dots & z_M \\ \vdots & \vdots & \dots & \vdots \\ z_1^{N-1} & z_2^{N-1} & \dots & z_M^{N-1} \end{bmatrix} \begin{bmatrix} R_1 \\ R_2 \\ \vdots \\ R_M \end{bmatrix}. \quad (44)$$

Considering the presence of noise, a data matrix $\mathbf{Y}$ is constructed based on $\mathbf{Y}_1$ and $\mathbf{Y}_2$:

$$\mathbf{Y} = \begin{bmatrix} y_0 & y_1 & \dots & y_L \\ y_1 & y_2 & \dots & y_{L+1} \\ \vdots & \vdots & \dots & \vdots \\ y_{N-L-1} & y_{N-L} & \dots & y_{N-1} \end{bmatrix}, \quad (45)$$

In order to obtain a filtered matrix, it is necessary to apply the Singular Value Decomposition (SVD) in the data matrix as presented by Eq. (46).

$$\mathbf{Y} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^H, \quad (46)$$

where $\mathbf{U}$ and $\mathbf{V}$ correspond to the eigenvectors of $\mathbf{Y} \mathbf{Y}^H$ and $\mathbf{Y}^H \mathbf{Y}$, respectively, $[\cdot]^H$ is the conjugate transpose operation, and $\mathbf{\Sigma}$ is the diagonal matrix of the singular values. Given the order M (see more details in Sheshyekani et al. (2017)), a filtered matrix $\check{\mathbf{V}}$ is constructed according to Eq. (47).

$$\check{\mathbf{V}} = [\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_M]. \quad (47)$$

Removing the last row and the first row of $\check{\mathbf{V}}$, the matrices $\check{\mathbf{V}}_1$ and $\check{\mathbf{V}}_2$ are obtained, respectively. The matrices are related with $\mathbf{Y}_1$ and $\mathbf{Y}_2$ by the following expressions:

$$\mathbf{Y}_1 = \mathbf{U}\check{\Sigma}\check{\mathbf{V}}_1, \quad (48)$$

$$\mathbf{Y}_2 = \mathbf{U}\check{\Sigma}\check{\mathbf{V}}_2, \quad (49)$$

where $\check{\Sigma}$ is the matrix of the singular values considering just the M filtered singular values.

Finally, the z_i parameters considering noise are obtained by the computing of the eigenvalues of Eq. (43), which is equivalent to the eigenvalues calculation: $\text{eig}(\mathbf{V}_1^+ \mathbf{V}_2)$, where $\text{eig}(\cdot)$ represents the eigenvalue calculation. The pseudo-code presented in the Pseudo-code 1 represents the steps executed by the MPM algorithm:

Pseudo-code 1 MPM algorithm

- 1: **Input:** $\mathbf{y} \in \mathbb{R}^N, N \in \mathbb{N}, L \in \mathbb{N}, M \in \mathbb{N}$
 - 2: Compute $\mathbf{Y}$ using (45)
 - 3: Apply SVD by computing $\mathbf{Y} \leftarrow \mathbf{U}\check{\Sigma}\mathbf{V}^H$
 - 4: Compute $\check{\mathbf{V}}$ using (47)
 - 5: Compute $\check{\mathbf{V}}_1 \leftarrow \check{\mathbf{V}}(1 : M - 1)$
 - 6: Compute $\check{\mathbf{V}}_2 \leftarrow \check{\mathbf{V}}(2 : M)$
 - 7: Obtain z_i computing $\text{eig}(\check{\mathbf{V}}_1^+ \check{\mathbf{V}}_2)$ ▷ $\text{eig}(\cdot) \rightarrow$ Eigenvalue calculation
 - 8: Construct $\mathbf{Z}$ of (44)
 - 9: Compute $\mathbf{s}$ using $\mathbf{s} \leftarrow \ln(\mathbf{Z})$
 - 10: Compute $\mathbf{R}$ using (44)
 - 11: Obtain $\mathbf{A}$ and φ by computing $\mathbf{A} = \sqrt{\text{Re}(\mathbf{R})^2 + \text{Im}(\mathbf{R})^2}$ and $\varphi = \arctan(\frac{\text{Im}(\mathbf{R})}{\text{Re}(\mathbf{R})})$
 - 12: Obtain ζ and ω by computing $\text{Re}(\mathbf{s})$ and $\text{Im}(\mathbf{s})$
 - 13: **Result:** $\mathbf{A} \in \mathbb{R}^M, \varphi \in \mathbb{R}^M, \omega \in \mathbb{R}^M, \zeta \in \mathbb{R}^M$
-

In the presented pseudo-code, line 2 is responsible for the data vector initialization. After the initialization, line 3 carries out the SVD of the data matrix to obtain the matrices $\check{\mathbf{V}}_1$ and $\check{\mathbf{V}}_2$. The complex-values z that form $\mathbf{Z}$ matrix are calculated by the eigenvalues (line 7) of the decomposed matrix. The matrix $\mathbf{Z}$ is constructed in line 8 and used to compute the $\mathbf{s}$ parameter in line 9 and the matrix $\mathbf{R}$ in line 10. Finally the amplitude, phase, damping factor and frequency are computed in lines 11 and 12, and returned as result in line 13.

The MPM has been proved an adequate technique for estimating harmonics and interharmonics in the power system (SHESHYEKANI et al., 2017). It achieves high-resolution,

and it does not work with a grid of frequencies, which prevents the spectral leakage. However, it presents a high computational load of $\mathcal{O}(N^3)$.

3.3 MATCHING PURSUIT ALGORITHM

The Matching Pursuit (MP) algorithm was first proposed by Mallat e Zhang (1993) and aims to decompose any signal into a linear combination of waveforms previously conceived by a dictionary of waveforms Φ . The algorithm expands the function into a set of waveforms selected from a dictionary, a better explanation about dictionaries will be presented in the next subsection. The dictionary is composed by atoms $\phi \in \mathbb{R}^N$, and $\|\phi\| = 1$, this way MP searches the atom that best matches with the signal by decomposing the signal $\mathbf{y} \in \mathbb{R}^N$ into the expression presented in Eq. (50),

$$\mathbf{y} = \langle \mathbf{y}, \phi_0 \rangle \phi_0 + \mathbf{r}, \quad (50)$$

where $\mathbf{r} \in \mathbb{R}^N$ represents the residue after the approximation of $\mathbf{y}$ into ϕ_0 direction. The orthogonality of the atom to $\mathbf{r}$ is assumed, hence:

$$\|\mathbf{y}\|^2 = \|\langle \mathbf{y}, \phi_0 \rangle\|^2 + \|\mathbf{r}\|^2. \quad (51)$$

The MP algorithm is an iterative procedure that minimizes the residue $\mathbf{r}$ by decomposing it through projections on the dictionary to select the atom that best matches the $\mathbf{r}$, considering the residue of the first iteration equivalent to the signal $\mathbf{r}_0 = \mathbf{y}$. It is noticed that although the MP is a non-linear algorithm, the energy is conserved throughout the entire process, this guarantees its convergence (MALLAT; ZHANG, 1993).

3.3.1 Dictionary

A dictionary Φ can be defined as a set of simple waveforms (atoms ϕ_γ) used to decompose the signals (RUBINSTEIN et al., 2010). The dictionary may form a *basis*; in this case, a linear combination of the atoms can represent an arbitrary signal. The index γ vary depending on the used dictionary (CHEN et al., 2001). Several dictionaries can be constructed in order to signal decomposition, here will be presented two main examples: Fourier dictionary (*Frequency dictionary*) and Wavelet dictionary (*Time-Scale dictionary*).

- Fourier dictionary

The Fourier dictionary (CHEN et al., 2001) is the main example of a *frequency dictionary* and its atoms ϕ_γ consist of sines and cosines, where $\gamma = (\omega, \nu)$, $\omega \in [0, 2\pi)$ being

the frequency and ν , either 0 or 1, is the indicator of the phase (sine or cosine). The Fourier atom is presented in Eq. (52)

$$\phi_{\omega,0} = \cos(\omega t) \quad \text{and} \quad \phi_{\omega,1} = \sin(\omega t). \quad (52)$$

The Fourier dictionary is suitable for stationary signal analysis and allows the decomposition of signals with Δf resolution, which is the ratio between the sampling frequency f_s and the number of samples N . This resolution can be refined by using an *l-fold overcomplete* dictionary. In this case, the frequency resolution Δf is improved l times, considering $l \in \mathbb{N}$ and $l > 1$.

- Wavelet dictionary

The Wavelet dictionary (AKANSU; HADDAD, 2001) is presented as an example of *time-scale dictionary* and is based on the Wavelet Transform (WT). The atoms are constructed based on a *wavelet mother* ψ and $\gamma = (a, b)$, where a is a positive value correspondent to the scale parameter and b is the localization parameter. The wavelet atom is presented in Eq. (53)

$$\phi_{a,b} = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right). \quad (53)$$

The atoms of this dictionary are based on the transformations performed in the wavelet mother ψ based on γ . It is an appropriated dictionary regarding the non-stationary signals and transient analysis.

In the following subsections two techniques that use MP theory are presented: the HIESSD and FMP.

3.4 HARMONICS AND INTERHARMONICS COMPONENTS ESTIMATION BASED ON SIGNAL SPARSE DECOMPOSITION

The HIESSD algorithm (PRADO et al., 2019) can estimate harmonics and interharmonics components in the power system signals using different levels of resolution in the frequency grid. The algorithm is based on the SSD technique and assumes that signals with harmonic and interharmonic distortions are formed by a limited number of frequency components and can be represented sparsely in the frequency domain. Considering its characteristics, the HIESSD is treated as a SSD problem that uses an *overcomplete hybrid dictionary* (OHD).

The SSD approach aims to represent a signal with components that compound a dictionary. The dictionary $\Phi \in \mathbb{R}^{N \times M}$ is formed by atoms, which consist of simple waveforms with different characteristics. A linear combination of the atoms may represent any arbitrary

signal $\mathbf{y} \in \mathbb{R}^N$ in time domain. Eq. (54) shows the principle of SSD:

$$\mathbf{y} = \sum_{m=1}^M \alpha_m \phi_m = \Phi \alpha, \quad (54)$$

where $\alpha \in \mathbb{R}^M$ is the vector of sparse coefficients resulting from the signal decomposition, α_m is a coefficient of the vector α and, ϕ_m corresponds to an atom in the dictionary.

The overcomplete dictionaries present $M > N$. In other words, the dictionary presents non-orthogonal atoms and redundant information. These features lead to robustness in the decomposition process (PRADO et al., 2019). Prado et al. (2019) treats the coefficients estimation as an optimization problem represented by the objective function in Eq. (55):

$$\hat{\alpha} = \min_{\alpha} \|\alpha\|_0 \quad \text{subject to} \quad \mathbf{y} = \Phi \alpha, \quad (55)$$

where $\|\cdot\|_0$ is the norm ℓ_0 which counts nonzero elements in a vector (TROPP et al., 2010). Considering the presence of noise, Prado et al. (2019) relax the problem as presented by Eq. (56):

$$\hat{\alpha} = \min_{\alpha} \|\alpha\|_0 \quad \text{subject to} \quad \|\mathbf{y} - \Phi \alpha\|_2 \leq \epsilon, \quad (56)$$

where $\epsilon > 0$ represents an error tolerance (TROPP et al., 2010). In order to search the solution for the problem presented in Eq. (56), HIESSD uses the *Orthogonal Matching Pursuit* (OMP) algorithm.

The HIESSD consists of two stages. Firstly, the algorithm builds the OHD considering the number of atoms for each signal, the type of the DTT basis (β), the number of samples (N), and the factor used to increase the grid (l). Prado et al. (2019) used the DCT/DST II pair to compose the OHD. The second stage consists of harmonics and interharmonics extraction. In this stage, four steps are performed: (I) the obtaining of the coefficients of the sparse vector; (II) the correction of the coefficients according to the used dictionary; (III) the calculation of the frequency components; and (IV) the application of a hard-threshold in order to eliminate the components resulting from the noise.

The OMP algorithm is performed as presented by Elad (2010):

1. The iterations counter k .
2. The vector of sparse coefficients $\hat{\alpha}$.
3. The residue $\mathbf{r}_k = (\mathbf{y} - \Phi \hat{\alpha})$.
4. Set $\mathcal{S}_k$ which represents the solution support $\hat{\alpha}$ (it contains the indices of the columns Φ which form the signal $\mathbf{y}$).

At each iteration, the OMP algorithm searches in ϕ the atom ϕ_m that results in the best approximation of the residue, which is done by calculating the inner product between the $\mathbf{r}_k$ and all atoms of the dictionary. The resulting index is stored in $\mathcal{S}_k$, and the coefficient related to that atom is calculated by $\hat{\alpha}_{k+1} = \arg \min_{\alpha} \|\mathbf{y} - \Phi_{\mathcal{S}_{k+1}} \hat{\alpha}\|$. The coefficient value is put in $\hat{\alpha}$, and $\mathbf{r}_{k+1}$ is updated. The process is repeated until the stop criterion $\|\mathbf{r}_k\| \leq \epsilon$ is reached.

The OMP algorithm carries out the decomposition of the signal $\mathbf{y}$ considering the $\mathbf{DCT}_{II}$ and $\mathbf{DST}_{II}$. However, the HIESSD performs a compensation considering the time shift in DCT-II, the Ψ_j and Γ_j are then calculated, according to the Eqs. (57) and (58):

$$\Gamma_j = \sqrt{\frac{2}{N}} (c_j \cos(\frac{j\pi}{2N}) + s_j \sin(\frac{j\pi}{2N})), \quad (57)$$

$$\Psi_j = \sqrt{\frac{2}{N}} (s_j \cos(\frac{j\pi}{2N}) - c_j \sin(\frac{j\pi}{2N})), \quad (58)$$

where the vectors $\mathbf{c}$ and $\mathbf{s}$ are $[c_0, c_1, \dots, c_{N-1}]^T$ and $[s_0, s_1, \dots, s_{N-1}]^T$, respectively, and are related to the atoms of the $\mathbf{DCT}_{II}$ and $\mathbf{DST}_{II}$, considering the coefficients vector $\hat{\alpha} = [\mathbf{c} \mid \mathbf{s}]^T$. Next the amplitude and phase of each component are calculated for Eqs. (59) and (60):

$$A_j = \sqrt{\Gamma_j^2 + \Psi_j^2}, \quad (59)$$

$$\varphi_j = \arctan\left(\frac{-\Psi_j}{\Gamma_j}\right). \quad (60)$$

Finally, the last step in the HIESSD algorithm is the removal of the influence of white Gaussian noise on the signal. This is achieved by applying a hard-threshold to the frequency components. The algorithm representing the HIESSD is presented by the pseudo-code 2.

Pseudo-code 2 HIESSD algorithm

```

1: Input:  $\Phi \in \mathbb{R}^{N \times M}$ ,  $\mathbf{y} \in \mathbb{R}^N$ ,  $\epsilon \in \mathbb{R}$ 
2: Initialize  $\mathbf{r}_k \leftarrow \mathbf{y}$ 
3: while  $\|\mathbf{r}_k\|_2 > \epsilon$ : ▷ begin OMP
4:   Compute  $\gamma_k \leftarrow \arg \max_m |\langle \mathbf{r}_k, \phi_m \rangle|$ 
5:   Update  $\mathcal{S}_{k+1} \leftarrow \mathcal{S}_k \cup \gamma_k$ 
6:   Compute  $\hat{\alpha}_{k+1} \leftarrow \arg \min_{\alpha} \|\mathbf{y} - \phi_{\mathcal{S}_{k+1}} \hat{\alpha}\|_2$ 
7:   Update  $\mathbf{r}_{k+1}$  by computing  $\mathbf{y} - \phi \hat{\alpha}_{k+1}$ 
8:   Update  $k \leftarrow k + 1$ 
9: end while ▷ end OMP
10: Compute  $\Gamma \leftarrow \sqrt{\frac{2}{N}}(c_j \cos(\frac{j\pi}{2N}) + s_j \sin(\frac{j\pi}{2N}))$ 
11: Compute  $\Psi \leftarrow \sqrt{\frac{2}{N}}(s_j \cos(\frac{j\pi}{2N}) - c_j \sin(\frac{j\pi}{2N}))$ 
12: Obtain  $\mathbf{A}$  and  $\varphi$  by computing  $\mathbf{A} = \sqrt{\Gamma_j^2 + \Psi_j^2}$  and  $\varphi = \arctan\left(\frac{-\Psi_j}{\Gamma_j}\right)$ 
13: Result:  $\mathbf{A} \in \mathbb{R}^M$ ,  $\varphi \in \mathbb{R}^M$ 

```

The pseudo-code presented the steps performed by HIESSD using OMP algorithm (lines 2-9). Line 2 corresponds to the initialization of the residue. The iteration process starts in line 3 and occurs until the residue achieves an acceptable value. Line 4 selects the atom index that results the maximum inner product with the residue. The coefficients with the minimum error between the signal and its reconstruction are selected in line 6. The residue is updated in line 7, in which the contribution of the atom selected is eliminated and the iteration counter is increased in line 8. At the end of the iterations the vector of coefficients $\hat{\alpha}$ is obtained, which allows the correction of the coefficients in lines 10 and 11. Finally, the amplitude and phase are calculated in line 12, respectively, and returned as result in line 13.

3.5 FAST MATCHING PURSUIT

The Fast Matching Pursuit is a method proposed by Lei et al. (2017). It is a technique based on SSD, which allows the estimation of harmonics and interharmonics components of a signal by employing iterative decomposition with an overcomplete sinusoidal dictionary. The method can achieve high-resolution results, even under noisy conditions. As HIESSD, the FMP considers a signal represented by Eq. (54):

$$\mathbf{y} = \sum_{m=1}^M \alpha_m \phi_m = \Phi \alpha.$$

and treats the coefficients estimation as an optimization problem, also represented by Eq. (55):

$$\hat{\alpha} = \min_{\alpha} \|\alpha\|_0 \quad \text{subject to} \quad \mathbf{y} = \Phi\alpha.$$

The main difference between the HIESSD and FMP methods consists in the used dictionary and technique used to solve Eq. (55). The FMP method uses MP technique for solving the problem. MP searches the atom ϕ_γ that matches best with the signal $\mathbf{y}$, as presented by Eq. (61):

$$\mathbf{y} = \langle \mathbf{y}, \phi_\gamma \rangle \phi_\gamma + \mathbf{r}, \quad (61)$$

where $\mathbf{r}$ is the residual after the iteration. The dictionary is formed by sinusoidal waveforms ϕ_γ , as presented in Eq. (62)

$$\phi_\gamma = A \cos(\omega t + \varphi), \quad (62)$$

where φ represents the phase, ω corresponds to the frequency, A is the normalized amplitude and γ corresponds to the parameters set (ω, φ) . FMP considers a discretized sub-dictionary and the atoms are discretized as presented in Eq. (63).

$$\gamma = (\omega, \varphi) = (k2^{-j}\pi, i\Delta\varphi), \quad (63)$$

where $\Delta\varphi = \frac{\pi}{6}$, $0 \leq k < 2^{j+1}$, $0 \leq i \leq 12$, $0 < j \leq \log_2 N$ and N is the number of samples.

It is desirable to minimize the residue, so it is necessary maximize the inner product $\langle \mathbf{y}, \phi_\gamma \rangle$, since ϕ_γ is orthogonal to $\mathbf{r}$ and the energy of the signal equal to the sum of the energies of the residue and ϕ_γ . Thus, the signal $\mathbf{y}$ can be decomposed in a sum of the orthogonal projections of the atoms in the dictionary. The atoms are selected iteratively assuming $\mathbf{r}_0 = \mathbf{y}$ and each step of the iterative algorithm is represented by Eq. (64), also the decomposed residue is represented by Eq. (65):

$$\phi_{\gamma_k} = \arg \max_{\phi_\gamma} |\langle \mathbf{r}_k, \phi_\gamma \rangle|, \quad (64)$$

$$\mathbf{r}_{k+1} = \langle \mathbf{r}_k, \phi_{\gamma_k} \rangle \phi_{\gamma_k} + \mathbf{r}_k. \quad (65)$$

In order to search for an adequate atom, Eq. (64) can be treated as a non-linear problem and an optimization algorithm can be applied to solve this problem. The FMP uses the Nelder-Mead algorithm (NMA) (SINGER; NELDER, 2009) with objective function

represented by Eq. (66), for estimating the frequency $\hat{\omega}$ and phase $\hat{\phi}$

$$F(\hat{\omega}, \hat{\phi}) = -|\langle \mathbf{r}_k, \phi_\gamma \rangle|. \quad (66)$$

Considering the process up to order M , the signal $\mathbf{y}$ can be represented as Eq. (67)

$$\mathbf{y} = \left(\sum_{m=0}^{M-1} \langle \mathbf{y}_m, \phi_{\gamma m} \rangle \phi_{\gamma m} \right) + \mathbf{r}_M. \quad (67)$$

The pseudo-code 3 shows the process to implement the FMP algorithm.

Pseudo-code 3 FMP algorithm

- 1: **Input:** $\mathbf{y} \in \mathbb{R}^N, N \in \mathbb{N}, \epsilon \in \mathbb{R}$
 - 2: Initialize $\mathbf{r}_0 \leftarrow \mathbf{y}$
 - 3: **while** $\|\mathbf{r}_k\| > \epsilon$:
 - 4: Compute initial ω_0 using $\gamma(\omega_0, \cdot) \leftarrow \arg \max_{\gamma(\omega, \cdot)} \|\mathcal{F}(\mathbf{r}_k)\|$ $\triangleright \mathcal{F}(\cdot) \rightarrow$ Fourier Transform
 - 5: Compute initial φ_0 using $\gamma(\omega_0, \varphi_0) \leftarrow \arg \max_{\gamma(\omega_0, i\Delta\varphi)} \langle \mathbf{r}_k, \phi_\gamma \rangle$
 - 6: Compute $(\hat{\omega}_k, \hat{\phi}_k)$ using $(\hat{\omega}_k, \hat{\phi}_k) \leftarrow \text{NMA}(\omega_0, \varphi_0)$
 - 7: Obtain A_k by computing $\langle \mathbf{r}_k, \phi_{\gamma(\omega_k, \phi_k)} \rangle$
 - 8: Compute $\phi_{\gamma k}$ using $A_k \cdot \phi_{\gamma(\omega_k, \phi_k)}$
 - 9: Update $\mathbf{r}$ by computing $\mathbf{r}_k \leftarrow \mathbf{r}_{k-1} - \phi_{\gamma k}$
 - 10: Update $k \leftarrow k + 1$
 - 11: **end while**
 - 12: **Result:** $\mathbf{A} \in \mathbb{R}^M, \boldsymbol{\varphi} \in \mathbb{R}^M, \boldsymbol{\omega} \in \mathbb{R}^M$
-

The presented algorithm shows the steps performed by the FMP algorithm. Line 2 shows the residue initialization. The loop is started in line 3 and occurs until the residue achieves an appropriate error level. The maximum energy frequency component is selected in line 4, where $\mathcal{F}(\cdot)$ refers to the Fourier Transform. Line 5 selects the initial value for the phase, according the discretized dictionary. The initial values of frequency and phase are used to initialize the Nelder-Mead algorithm, which finds the optimal values in line 6. The amplitude value is then found by relating the atom formed by the frequency and phase values previously found in line 7. The formed atom is computed in line 8. Finally, the residue and the iterations counter are updated in lines 9 and 10, respectively and the amplitude, frequency and phase vectors are returned as result in line 12.

3.6 CHAPTER REVIEW

The state-of-art methods for extraction of harmonics and interharmonics components were presented in detail in this Chapter: the MPM, the MP, which provided theoretical base to the HIESSD, and the FMP algorithms.

4 PROPOSED METHOD

This chapter presented the theoretical foundation of the ALMP algorithm and the steps performed for the signal decomposition. First, a signal modeling and dictionary design are presented in order to establish essential concepts used by the algorithm. Then, the Anti-Leakage Matching Pursuit is presented in details: an initial parameters selection occurs, after an optimization step is responsible for the atom selection, then the residue and the reconstructed signal are updated and the residue is used to verify the stop criteria of the algorithm.

4.1 SIGNAL MODELING

The techniques based on Matching Pursuit (MALLAT; ZHANG, 1993) assume that signals with harmonics and interharmonics are composed by a limited number of waveforms, which allows the decomposition of the signal into a sum of predefined waveforms, the atoms $\phi \in \mathbb{R}^N$, that form a dictionary $\Phi \in \mathbb{R}^{N \times M}$. Based on the MP technique, the ALMP algorithm considers the signal as a linear combination of atoms. The atoms are formed based a non-fixed Fourier dictionary and as the iterations occur they are used to compound the dictionary, as presented by Eq. (68)

$$\mathbf{y} = \sum_{k=1}^K \phi_k + \boldsymbol{\eta}, \quad (68)$$

where $\mathbf{y} \in \mathbb{R}^N$ is an arbitrary signal, the iterations occur until K iterations, K should be equal to M , the order of the signal, i.e., the number of harmonics and interharmonics that compound the signal and $\boldsymbol{\eta} \in \mathbb{R}^N$ represents the white Gaussian noise. Note that a previous knowledge of the order of the signal is not necessary, since the algorithm is iterative and designed to stop at M iterations as the residue will be minimum at this point.

4.2 DICTIONARY DESIGN

We propose the use of an iterative Fourier dictionary for the atoms selection. Unlike the other methods of decomposition presented, the ALMP algorithm assumes a dictionary that is not fixed or predefined. The atoms are assembled as long as the parameters of amplitude,

phase and frequency are estimated by the ALMP algorithm. We use sinusoidal atoms, as shown in Eq. (69).

$$\phi_\gamma(n) = A \cos(\omega n + \varphi), \quad (69)$$

where $\gamma = [A, \omega, \varphi]^T$, A is the normalized amplitude, ω corresponds to the angular frequency, φ represents the phase and $n = 0, 1, 2, \dots, N - 1$. The choice of the dictionary was based in the ALMP characteristics, in which the components are estimated according to the Fourier Transform of the signal.

As mentioned above, any arbitrary signal may be expressed as a linear combination of sinusoidal atoms. This principle is illustrated by Figure 12, which shows a distorted signal decomposed as the sum of three other atoms formed by different parameters set. The signal was sampled according to the IEC 61000-4-7 and it represents the signal from an overexcited transformer presented in Eq. (1). In this example the non-fixed dictionary, assembled as the iterations occur, considers just the selected atoms ϕ_{γ_1} , ϕ_{γ_2} and ϕ_{γ_3} .

Figure 12: A distorted signal represented by a linear combination of atoms.

4.3 PARAMETERS ESTIMATION WITH ALMP

The ALMP algorithm is able of estimating the harmonics and interharmonics components of power system signals by using the MP theory. The atoms selection is given by the establishment of a non-linear least square problem that aims to minimize the energy of the residue in order to find the atom that best matches the residue at each iteration. The formulated optimization problem is presented in Eq. (70):

$$\hat{\gamma}_k = \arg \min_{\gamma} ||\mathbf{r}_k - \phi_\gamma||^2, \quad (70)$$

where $\hat{\gamma}_k$ correspond to the parameters set $(\hat{A}_k, \hat{\omega}_k, \hat{\varphi}_k)$, being the amplitude, frequency and phase, respectively, of the current iteration k , ϕ_γ corresponds to the atom presented in Eq. (69) and $\mathbf{r}_k$ is the residue. As the MP algorithm, at each iteration the ALMP algorithm searches the atom that best match with the signal by computing Eq. (70), which may be solved by optimization algorithms for non-linear least-squares problems. We considered three algorithms

suitable for this type of problem and presented in recent papers: the Levenberg-Marquadt algorithm, Nelder-Mead algorithm and Quasi-Newton method, in the next subsection they will be presented in more details.

The Eq. (70) express the main step of the ALMP algorithm. The problem is formulated in order to select the parameters set to compound the atom that minimizes the energy of the residue, i.e., the contribution of the atom is subtracted from the residue signal, and the parameters set that minimizes the energy resulting from this operation is the parameters set selected to compound the final atom. The establishment of this problem provides the robustness to noise and deviations of the algorithm.

At the first iteration $\mathbf{r}_0 = \mathbf{y}$, and at each iteration, an initial estimate for the parameters set is performed. Given the Fourier Transform of the residue $\mathcal{F}(\mathbf{r}_{k-1})$, the initial parameters selection occurs based on the atom that satisfies Eq. (71):

$$\phi_{\gamma_{0_k}} = \|\mathcal{F}(\mathbf{r}_k)\|_{\infty}. \quad (71)$$

The initial atom $\phi_{\gamma_{0_k}}$ computed by Eq. (71), used as initial point for the optimization algorithm, is composed by the initial parameter set $[A_{0_k}, \omega_{0_k}, \varphi_{0_k}]^T$. Then, the parameters set is adjusted in the optimization stage to obtain final values to compound the atom, as shown in Eq. (70).

At the end of the iteration, the atom selected by Eq. (70) has its contribution subtracted from the residue of the signal ($\mathbf{r}_k \in \mathbb{R}^n$) in order to update the residue by removing the influence of the estimated component as presented by Eq. (72),

$$\mathbf{r}_{k+1} = \mathbf{r}_k - \phi_{\hat{\gamma}_k}. \quad (72)$$

In addition, the estimated component is added in the partial reconstructed signal $\mathbf{x} \in \mathbb{R}^N$, as shown in Eq. (73).

$$\mathbf{x}_{k+1} = \mathbf{x}_k + \phi_{\hat{\gamma}_k}, \quad (73)$$

note that at $k = M$, the signal $\mathbf{x}$ corresponds to the reconstructed signal at this point and should be equal or similar to the original signal, $\mathbf{x} \approx \mathbf{y}$.

The process is repeated until the energy of the residue cease to decrease as the iterations occurs. Eq. (74) presents this stage of verification.

$$\|\mathbf{r}_k\|^2 - \|\mathbf{r}_{k+1}\|^2 \leq \epsilon, \quad (74)$$

where the tolerance value is represented by ϵ and it is determined according to the variance of the harmonics and interharmonics components. A flowchart presenting the ALMP algorithm is shown in Figure 13(a) and an illustrative example of the steps is presented in Figure 13(b).

(a) Flowchart representing the steps performed by the ALMP Algorithm. (b) Illustrative example of the ALMP algorithm.

Figure 13: Steps performed by the ALMP algorithm.

The Figure 13 illustrates the algorithm steps. Figure 13(b) allows a graphic visualization of one iteration of the ALMP algorithm. First, a distorted signal (**I**) is considered and a Fourier Transform is applied on it, the greater component resulting from this process is selected (**II**) and the parameters that compound this component are used to form the initial parameters set γ_0 . This parameters set is used as initial point for an optimization algorithm (**III**), that selects the optimum parameters set $\hat{\gamma}$. The set $\hat{\gamma}$ compound the selected atom, that is used to update the residue and the reconstructed signal (**IV**). If the stop criteria is achieved the algorithm stops, Otherwise, it returns to **I**. The pseudo-code 4 represents the ALMP algorithm:

Pseudo-code 4 ALMP algorithm

```

1: Input:  $\mathbf{y} \in \mathbb{R}^N$ ,  $N \in \mathbb{N}$ ,  $\epsilon \in \mathbb{R}$ 
2: Initialize  $\mathbf{r}_0 \leftarrow \mathbf{y}$ ,  $k \leftarrow 0$ 
3: while 1:
4:   Compute initial  $\gamma_0$  using  $\phi_{\gamma_0} \leftarrow \|\mathcal{F}(\mathbf{r}_k)\|_\infty$ 
5:   Compute  $\hat{\gamma}_k(\hat{A}_k, \hat{\omega}_k, \hat{\varphi}_k) \leftarrow \arg \min_{\gamma} \|\mathbf{r}_k - \phi_\gamma\|^2$ 
6:   Obtain  $\phi_{\hat{\gamma}_k}$  computing  $\hat{A}_k \cos(\hat{\omega}_k n + \hat{\varphi}_k)$ 
7:   Update  $\mathbf{r}$  by computing  $\mathbf{r}_{k+1} \leftarrow \mathbf{r}_k - \phi_{\hat{\gamma}_k}$ 
8:   Update  $\mathbf{x}$  by computing  $\mathbf{x}_{k+1} \leftarrow \mathbf{x}_k + \phi_{\hat{\gamma}_k}$ 
9:   if  $\|\mathbf{r}_k\|^2 - \|\mathbf{r}_{k+1}\|^2 \leq \epsilon$ 
10:    break
11:   else
12:     Update  $k \leftarrow k + 1$ 
13:   end if
14: end while
15: Result:  $\hat{A} \in \mathbb{R}^M$ ,  $\hat{\varphi} \in \mathbb{R}^M$ ,  $\hat{\omega} \in \mathbb{R}^M$ 

```

The presented pseudo-code shows all the steps performed by the ALMP algorithm. First, the residue and the counter parameter are initialized in line 2, then the loop starts in line 3. The initial parameters set is computed in line 4, used as initial point by an optimization algorithm in line 5, then provides an optimum parameters set to compound the selected atom in line 6. Finally, the contribution of the optimum atom is decreased from the residue in line 7 and added in the signal reconstruction in line 8. The tolerance verification is performed in line 9, if the tolerance is achieved the loop ends in line 10, otherwise the counter is updated in line 12 and the algorithm restarts. Finally, the vectors of amplitude, phase and frequency are returned as result in line 15.

4.4 CHAPTER REVIEW

This chapter presented the theoretical foundation of the ALMP algorithm. The steps performed by the algorithm are presented in detail, which consists in a first parameters estimation that provide a set of initial parameters to an optimization step, which selects the optimum atom, then the residue is updated, as well as the reconstructed signal. The stop criteria is analyzed and if the residue remains stable as the iterations occurs the algorithm stops, otherwise it returns to the first step.

5 RESULTS AND DISCUSSIONS

This chapter presents the results of the ALMP algorithm for the signals presented in this work, as well as a comparative analysis with methods in the literature considering the presented signals: (I) single-phase DC source (BOLLEN; GU, 2006); (II) six-pulse converter (RICE, 1994); (III) synchronous machine (WALLACE et al., 1974); (IV) overexcited transformer (ARRILLAGA; WATSON, 2003); (V) cycloconverter (SYAM et al., 2004); and (VI) microgrid (WANG et al., 2012). In order to evaluate the algorithm behavior considering IEC 61000-4-7 (IEC, 2006), all the signals presented in this chapter were sampled with a 200 ms sampling window. The sampling rate has been set at 256 samples per cycle which results in a sampling frequency of 15360 Hz and $N = 3072$. The MAE values presented on the results were obtained considering an average of 3 measurements and the algorithms were implemented using MATLAB software on a PC with Intel Core i7-6700 3.40 GHz Processor.

5.1 NON-LINEAR LEAST SQUARE ALGORITHM

The optimization algorithms for solving non-linear least square problems were evaluated and selected among the three presented algorithms: LMA, NMA and QN-BFGS. In order to perform a comparison between the methods, first an analysis of the error regarding the signal-noise ratio (SNR) was performed considering a pure sine wave as presented in Eq. (75),

$$y(t) = \cos(2\pi ft + 30^\circ), \quad (75)$$

where $y(t)$ is the signal, f is the fundamental frequency, considered 60 Hz in this case, and t is the time. Figure 14 presents the error considering the normalized frequency and amplitude in function of SNR, that varies between 10 dB to 100 dB. Whilst it is observed that the three algorithms presented very similar error rates, the QN-BFGS and LMA presented a lower error rate when compared to the NMA.

(a) Mean absolute error in amplitude estimation.

(b) Mean absolute error in frequency estimation.

(c) Mean absolute error in phase estimation.

Figure 14: Mean absolute error in estimation according to the SNR.

As observed in Figure 14 the performance regarding the algorithm accuracy, assessed by MAE, was similar between the algorithms. Considering the amplitude estimation, the algorithms presented differences in the order of 10^{-6} pu, for frequency estimation the estimations differed with the order of 10^{-5} Hz and regarding the phase estimation the difference between the algorithms presented the order of 10^{-4}° . Considering the other noise values, the algorithms behavior remained the same, as one can see in Table 1, the Table present only the errors obtained until 50 dB, since the errors become smaller, of the order of 10^{-7} , as the SNR increased.

Table 1: Comparative analysis between the optimization algorithms according to the SNR.

		10 dB	20 dB	30 dB	40 dB	50 dB
LMA	Amp. (pu)	3.997×10^{-3}	1.933×10^{-3}	3.571×10^{-4}	6.637×10^{-5}	7.170×10^{-5}
	Freq. (Hz)	7.992×10^{-3}	2.536×10^{-4}	1.721×10^{-3}	4.084×10^{-4}	2.047×10^{-5}
	Phase ($^\circ$)	6.090×10^{-1}	1.099×10^{-2}	8.045×10^{-3}	1.658×10^{-2}	1.840×10^{-3}
NMA	Amp. (pu)	3.998×10^{-3}	1.936×10^{-3}	3.594×10^{-4}	6.752×10^{-5}	7.144×10^{-5}
	Freq. (Hz)	7.998×10^{-3}	2.564×10^{-4}	1.726×10^{-3}	4.127×10^{-4}	1.517×10^{-5}
	Phase ($^\circ$)	6.091×10^{-1}	1.106×10^{-2}	8.058×10^{-3}	1.680×10^{-2}	1.819×10^{-3}
QN-BFGS	Amp. (pu)	3.997×10^{-3}	1.933×10^{-3}	3.571×10^{-4}	6.637×10^{-5}	7.181×10^{-5}
	Freq. (Hz)	7.993×10^{-3}	2.565×10^{-4}	1.720×10^{-3}	4.184×10^{-4}	1.105×10^{-5}
	Phase ($^\circ$)	6.091×10^{-1}	1.109×10^{-2}	7.983×10^{-3}	1.694×10^{-2}	1.112×10^{-3}

Since the algorithms presented similar behavior under noise conditions, a generic signal compound by the fundamental frequency of 60 Hz, one harmonic and one interharmonic, according to Eq. (76) was considered.

$$y(t) = \cos(2\pi ft + 30^\circ) + 0.3 \cos(2\pi 3ft - 10^\circ) + 0.1 \cos(2\pi 222t), \quad (76)$$

where the generic signal is represented by $y(t)$ and the fundamental frequency f is 60 Hz. The MAE, execution time and floating points operations (FLOPS) obtained by the three optimization algorithms were analyzed. The signal were computed considering a SNR of 40 dB, regarding the voltage limits of 1% established by the IEEE (2014) and the MAE was computed considering the average of the MAE obtained for each harmonic and interharmonic component estimation, the execution time was obtained as a result of the average of 10 samples and the FLOPS were obtained through the MATLAB function FLOPS() (QIAN, 2020). Table 2 presents the mean absolute errors obtained by the algorithms, as well as the execution time and FLOPS.

As observed in Figure 14 and in Table 2, the performance regarding the algorithm accuracy, assessed by MAE, was similar between the algorithms. However, the main difference between the algorithms concerns the execution time and FLOPS presented. As one can see,

Table 2: Comparative analysis between the optimization algorithms.

		LMA	NMA	QN-BFGS
MAE	Amp. ($\times 10^{-5}$ pu)	58.34	58.27	58.34
	Freq. ($\times 10^{-4}$ Hz)	32.50	32.52	32.51
	Phase ($\times 10^{-2}^\circ$)	32.45	32.45	32.46
Time (ms)		19.54	23.08	27.10
FLOPS($\times 10^4$)		13.52	15.06	15.06

although the accuracy is essentially the same for the three algorithms, the LMA presented the lower execution time and number of FLOPS, being 1.18 times faster than the NMA algorithm and 1.39 times faster than the QN algorithm. Also it performed about 15349 fewer floating points operations. For those reasons, the Levenberg-Marquadt algorithm was chose to compose the optimization stage of the ALMP.

5.2 ALGORITHM EVALUATION

The ALMP algorithm returns frequency, amplitude and phase information of each harmonic and interharmonic component present in the signal under noisy conditions even with fundamental frequency deviation. The analyzed signals are formed by harmonics and interharmonics components and the MAE is used to compare the methods in the literature. All the algorithms were implemented on MATLAB and the following methods were implemented and compared to the ALMP algorithm:

- DFT (implemented with FFT algorithm);
- MPM (SHESHYEKANI et al., 2017);
- FMP (LEI et al., 2017);
- HIESSD (PRADO et al., 2019).

The signals were considered according to its characteristics in order to compare the performance of the ALMP algorithm with the presented methods considering the following comparison criteria:

- (I) Single-phase DC source (BOLLEN; GU, 2006) - Harmonics estimation;
- (II) Six-pulse converter (RICE, 1994) - Robustness to noise in harmonics estimation;
- (III) Synchronous Machine (WALLACE et al., 1974) - Interharmonics Estimation;

(IV) Overexcited Transformer (ARRILLAGA; WATSON, 2003) - Robustness to frequency deviation;

(V) Cycloconverter (SYAM et al., 2004) - Interharmonics estimation of a less-complex signal under high-noisy conditions and fundamental frequency deviation;

(VI) Microgrid (WANG et al., 2012) - Interharmonics estimation of a more complex signal under fundamental frequency deviation conditions.

The signals considered the IEC 61000-4-7 (IEC, 2006) and were sampled with a 200 ms sampling window. The sampling rate has been set at 256 samples per cycle which results in a sampling frequency of 15360 Hz and $N = 3072$.

5.2.1 Single-phase DC source

In order to assess the correct performance of the algorithm to estimate the harmonics components under noise conditions, the supply current from a DC source was simulated and its components extract by the ALMP, DFT, HIESSD and FMP algorithms. The i_{DC} was considered equal to $1/(2\sqrt{2})$ to normalize the signal, and a white Gaussian noise with SNR=40 dB was considered. Recalling the Eq. (2) that models the supply current from a single-phase DC source:

$$i_h = \begin{cases} +\frac{2\sqrt{2}}{h}i_{DC} & h = 1, 5, 9, 13, \dots \\ -\frac{2\sqrt{2}}{h}i_{DC} & h = 3, 7, 11, 15, \dots \\ 0 & h = 0, 2, 4, 6, \dots \end{cases}$$

The results of the decomposition of this signal by the ALMP, HIESSD, DFT, MPM and FMP algorithms are shown in Figs. 15(a) and 15(b) considering the estimation of amplitude and phase, respectively.

(a) Mean absolute errors in estimation of amplitude.

(b) Mean absolute errors in the estimation of phase.

Figure 15: Estimation of the harmonic components of the supply current from a DC source added with a white Gaussian noise with SNR=40 dB. The harmonic components have been estimated by the DFT, MPM, FMP, HIESSD and ALMP algorithms.

Regarding the amplitude estimation, which the MAE is represented in the graphic in Figure 15(a), it is observed the DFT and the ALMP algorithms presented the lowest average values for the errors. The MPM algorithm also presented a good performance, although it presented errors in all harmonic components. The HIESSD algorithm presented the smallest error considering the components of greater energy. However, as the components decreased their energy the errors increased. The FMP algorithm also presented the higher values for the error in amplitude. In general, the tests showed an equivalence between the DFT and ALMP algorithms in terms of errors, as one can see in Figure 15. Regarding the phase estimation, the FMP and MPM presented the larger errors, while the DFT, HIESSD and ALMP algorithms presented the smaller errors. The ALMP presented the lowest average MAE for both amplitude and phase estimations.

In terms of amplitude, the ALMP algorithm presented an average MAE of 1.6×10^{-3} pu, while the DFT, MPM, FMP and HIESSD presented 1.8×10^{-3} pu, 4.3×10^{-3} pu, 8.3×10^{-3} pu and 8.3×10^{-3} pu, respectively. In terms of phase estimation, the ALMP algorithm also presented the lowest MAE of 0.06° , while the DFT, MPM, FMP and HIESSD presented 0.11° , 0.22° , 1.10° and 0.11° , respectively.

5.2.2 Six-pulse Converter

The six-pulse converter signal was analyzed in order to evaluate the ALMP performance under high-level noise conditions. The signal contain only harmonics components, as one can recall from the Eq. (3):

$$i_h = \frac{1}{h} i_1, \quad \text{for } h = 6j \pm 1 \text{ and } j = 1, 2, 3, \dots,$$

The frequency components of the signal and the MAE presented by the ALMP, HIESSD, DFT, MPM and FMP algorithms (under varied noise conditions) are presented in Figs. 16, 17 and 18. The MAE presented by the phase estimation performed by the HIESSD algorithm has been omitted from the graphics, since it presented large errors hindering the visualization of the MAE presented by the other algorithms.

The lower components are not identified by all the algorithms, as long the noise level is increased. It is possible to see that the HIESSD and DFT have estimated the frequency components limited by the tolerance value, which increased as the noise was raised leading to the non-estimation of the components with high frequency. Meanwhile, the MPM have removed frequency components with the amplitude greater than τ . As the noise level was raised, some components were not estimated, which increased the inaccuracy of the results. Although the FMP method identified the same frequency components as ALMP, HIESSD, DFT algorithms, the algorithm identified several non-existent components. A correct estimation was observed from ALMP algorithm. It presented robustness to noise and obtained values similar to DFT and HIESSD regarding the amplitude estimation.

Considering the phase estimation, the HIESSD algorithm did not present a satisfactory estimation, since it presented a lag of 180° in several components and estimations with large errors. The MPM algorithm even under low noise condition was not capable of estimating all the frequency components. The DFT and ALMP algorithms presented better results regarding the phase estimation under high-noise conditions. Table 3 summarizes the average errors presented by each algorithm under different noise conditions. Note that the non-existent components estimated by the FMP were considered for computing its average MAE.

(a) Mean absolute errors in estimation of amplitude.

(b) Mean absolute errors in the estimation of phase.

Figure 16: Performance of the DFT, MPM, FMP, HIESSD and ALMP algorithms for estimating harmonic components in the current of a six-pulse three-phase rectifier considering a noise rate of 40 dB. The components marked by (*) were not estimated by the MPM algorithm.

(a) Mean absolute errors in estimation of amplitude.

(b) Mean absolute errors in the estimation of phase.

Figure 17: Performance of the DFT, MPM, FMP, HIESSD and ALMP algorithms for estimating harmonic components in the current of a six-pulse three-phase rectifier considering a noise rate of 20 dB. The components marked by (*) were not estimated by the MPM algorithm and the components marked by () were not estimated by all the considered algorithms.**

(a) Mean absolute errors in estimation of amplitude.

(b) Mean absolute errors in the estimation of phase.

Figure 18: Performance of the DFT, MPM, FMP, HIESSD and ALMP algorithms for estimating harmonic components in the current of a six-pulse three-phase rectifier considering a noise rate of 10 dB. The components marked by (*) were not estimated by the MPM algorithm and the components marked by () were not estimated by all the considered algorithms.**

Table 3: Average errors presented by the DFT, MPM, FMP, HIESSD and ALMP algorithms for estimating harmonic components in the current of a six-pulse three-phase rectifier on different conditions of noise.

	DFT		MPM		FMP		HIESSD		ALMP	
	Amp. (pu)	Phase(°)	Amp. (pu)	Phase(°)	Amp. (pu)	Phase(°)	Amp. (pu)	Phase(°)	Amp. (pu)	Phase(°)
40 dB	0.0002	0.5827	0.0015	19.9977	0.0075	98.0484	0.0002	94.2483	0.0002	1.0751
20 dB	0.0028	21.4688	0.0189	58.2330	0.0213	114.4693	0.0029	92.7402	0.0020	23.4509
10 dB	0.0077	40.3616	0.0539	76.9367	0.0170	72.3309	0.0083	64.8853	0.0079	44.2704

As one can see, the DFT and ALMP algorithms presented similar estimations considering the amplitude and phase estimations. As shown in Figs. 16, 17 and 18, the values presented in Table 3 confirmed the large errors in the MPM estimations, which were 9.5 times larger than the ALMP for the amplitude and 2.5 times larger than the ALMP algorithm for the phase considering 20 dB. This is due to its susceptibility to noise. Also the HIESSD presented larger errors than the other algorithms regarding the phase estimations, its errors achieved 94.25° for 40 dB, while the ALMP presented 1.08°. Although the FMP having presented ac-

ceptable errors in the components that compound the signal as can be seen in Figures 16-18, it estimated several non-existent components with high energy, that is reflected in the average MAE of the algorithm in Table 3 and in the spectrum obtained by the algorithm, as shown in Figures 19(a)-19(c), where the fundamental frequency has been omitted for a better view. Considering 40 dB, the MAE obtained by the FMP was 0.0075 against 0.0015 obtained by the MPM, and 0.0002 obtained by the DFT, HIESSD and ALMP algorithms. Finally the DFT and ALMP algorithms presented similar performances, since the differences between the amplitude estimations presented the order of 10^{-4} pu and considering the phase estimation, the DFT performance was superior presenting a MAE up to 3.91° inferior than the ALMP algorithm.

(a) Spectrum obtained by the FMP algorithm considering a noise rate of 40 dB.

(b) Spectrum obtained by the FMP algorithm considering a noise rate of 20 dB.

(c) Spectrum obtained by the FMP algorithm considering a noise rate of 10 dB.

Figure 19: Spectrum obtained by the FMP algorithm considering varied noise conditions.

5.2.3 Synchronous Machine

The signal from synchronous machine was simulated in order to verify the ALMP behavior considering interharmonics presence. The signal is formed by only interharmonics components and the fundamental frequency, as presented in Eq. (5):

$$\begin{aligned} I_s(t) = & 0.03 \sin(2\pi 24t) + 0.024 \sin(2\pi 48t) + \sin(2\pi 60t) \\ & + 0.023 \sin(2\pi 96t) + 0.029 \sin(2\pi 264t) + 0.03 \sin(2\pi 384t) \\ & + 0.003 \sin(2\pi 588t) + 0.004 \sin(2\pi 708t). \end{aligned}$$

The frequency values present in this signal are not in the frequency grid of the DFT and HIESSD for $l = 1$. However, when we set $l = 5$, the frequency resolution of the HIESSD increases to 0.5 Hz. The MAE results for estimating the amplitude of the interharmonics components of the test signal with $\text{SNR} = 40$ dB using the MPM, FMP, HIESSD with frequency resolution of 0.5 Hz and the ALMP algorithms are shown in Figure 20(a). Also, the MAE in frequency estimation was evaluated and presented in Figure 20(b) considering the algorithms without fixed grid: the MPM, FMP and ALMP. The phase was not evaluated, since it is zero for all components.

According to Figure 20, the two smaller amplitude components are not identified by MPM, while the other components are estimated with errors in frequency and amplitude in the order of 10^{-2} and 10^{-4} , respectively. The FMP algorithm identified the eight components, but with larger errors than MPM. Among the implemented algorithms, the HIESSD presented the lower values of error. However, ALMP also identified all the components that compound the signal with lower errors when compared to FMP and the MAE in amplitude estimations are in the order of 10^{-4} .

Comparing the results, it is observed that the the ALMP and HIESSD algorithms estimate correctly all frequency components of the signal, while the MPM algorithm does not identify the two smaller components and the FMP presented larger errors than other methods. The HIESSD presented the average MAE of 1×10^{-4} pu in the amplitude estimation, followed by the ALMP algorithm, that presented an average of 5×10^{-4} of MAE, the MPM presented 11×10^{-4} and the FMP presented the larger MAE of 16×10^{-4} , a larger error than the MPM, which did not estimated two interharmonics. Considering the algorithms without a fixed grid, the ALMP presented the better performance for both amplitude and frequency estimations. It presented an average of 0.07 Hz of MAE in frequency estimation, while the FMP and MPM presented 0.19 and 162.02, respectively, this large value obtained by the MPM is due to the two interharmonics with high frequency that were not estimated by the algorithm.

(a) Mean absolute errors in the estimation of the amplitude without the fundamental component.

(b) Mean absolute errors in the estimation of the frequency by the non-fixed grid algorithms.

Figure 20: Frequency components estimation from the MPM, FMP, HIESSD and ALMP algorithms for the stator current of a synchronous machine. The components marked by (*) were not estimated by the MPM algorithm.

5.2.4 Overexcited Transformer

The signal from the overexcited transformer was evaluated considering 0.3 Hz of frequency deviation in the fundamental frequency and SNR = 40 dB. The signal is presented in Eq. (1) and contains two harmonic components:

$$y(t) = \sin(\omega t) - 0.18 \sin(3\omega t) + 0.11 \sin(5\omega t).$$

Table 4 shows the performance of MPM, FMP and ALMP algorithms in estimating the amplitudes and frequencies. It is observed that the three algorithms correctly estimated the components of the signal. The ALMP algorithm presented a performance similar to the MPM algorithm. The FMP algorithm, although performed a satisfactory estimation, presented more deviations when compared with the other two algorithms. The DFT and HIESSD are not presented in Tab. 4, since they do not provide support for signals with frequency deviation.

However, the performance of the algorithms is presented in Figure 21, in which the spectrum obtained from each algorithm is showed.

Table 4: Mean absolute errors from the estimation of the overexcited transformer signal considering a frequency deviation of 0.3 Hz. The components marked by (*) were not estimated by the MPM algorithm.

		40 dB			20 dB			10 dB		
Amp.(pu)	MPM	0.0005	0.0002	0.0003	0.0015	0.0004	0.0057	0.0045	(*)	(*)
	FMP	0.0043	0.0005	0.0026	0.0046	0.0038	0.0129	0.0159	0.0116	0.0091
	ALMP	0.0006	0.0006	0.0001	0.0003	0.0040	0.0038	0.0110	0.0117	0.0026
Freq.(Hz)	MPM	0.0002	0.0027	0.0033	0.0047	0.0521	0.0121	0.0017	(*)	(*)
	FPM	0.0591	0.0419	0.0260	0.0651	0.0085	0.0419	0.0581	0.0387	0.0258
	ALMP	0.0053	0.0349	0.0022	0.0114	0.0160	0.0303	0.0045	0.0309	0.0183
Phase(°)	MPM	0.0080	0.1254	0.0447	0.2042	0.8470	1.4968	0.1374	(*)	(*)
	FPM	2.2234	1.7258	1.0261	2.4726	0.8411	2.6996	2.0971	2.1651	2.7411
	ALMP	0.1812	1.2807	0.0082	0.4332	0.3836	2.1488	0.0735	1.6899	1.4255

Figure 21 shows the real frequency spectrum obtained by the algorithms. Figure 21a shows the real spectrum used as reference. Figure 21b and 21d show the spectrum obtained by the DFT and HIESSD algorithms, both spectra present *spectral leakage*, since they are algorithms with fixed grid and thus are not able to estimate signals with frequency deviation. The ALMP and MPM algorithms presented similar spectra, while FMP estimated several minor components that are not presented in the signal.

(a) Reference Spectrum.

(b) Spectrum resulting from DFT algorithm.

(c) Spectrum resulting from MPM algorithm.

(d) Spectrum resulting from HIESSD algorithm with resolution of 2.5 Hz.

(e) Spectrum resulting from FMP algorithm.

(f) Spectrum resulting from ALMP algorithm.

Figure 21: The estimated frequency spectra resulting from the signal decomposition of the current of the overexcited transform with $\text{SNR} = 40$ dB.

5.2.5 Cycloconverter

The signal from the output of a cycloconverter was simulated in order to verify the estimations performed by the algorithm under frequency deviation and interharmonics presence. Considering the signal obtained experimentally by Syam et al. (2004), the cycloconverter input, besides the fundamental frequency of 50.75 Hz, presents two interharmonics in 3.5 Hz and 98 Hz, as previously showed in Eq. (4):

$$y(t) = 0.66 \sin(2\pi f_1 t) + 0.48 \sin(2\pi(2f_1 - 3f_o)t) + 0.31 \sin(2\pi(4f_1 - 3f_o)t).$$

The frequencies that compound the input signal from a cycloconverter can not be estimated by the DFT and HIESSD algorithms, since they are non-multiple of 5 Hz and 0.5 Hz, which is the resolution achieved by the HIESSD algorithm. The signal was submitted to a noise rate of 10 dB and the estimation of its components was performed by the MPM, FMP and ALMP algorithms. The phase estimation was not considered since Syam et al. (2004) did not present the phase values. Figure 22 presents the spectrum obtained by the MPM, FMP and ALMP algorithms. Figures 22(b)-(d) present the individual components that form the signal, 3.5 Hz, 50.75 Hz and 98 Hz, respectively.

(a) Estimation of Cycloconverter Input Signal.

(b) Estimation of the 3.5 Hz component.

(c) Estimation of the 50.75 Hz component.

(d) Estimation of the 98 Hz component.

Figure 22: Estimation of Cycloconverter Input Signal by the MPM, FMP and ALMP algorithms.

It is observed that the MPM and ALMP algorithms presented similar results, since they were the algorithms more similar to the reference. This may be concluded when the mean error presented by each algorithm is observed: the ALMP algorithm presented the lowest average error for both amplitude and frequency estimations, which are, respectively, 6.8×10^{-3} pu and

42.2×10^{-3} Hz, the MPM presented errors equal to 8.1×10^{-3} pu and 46.3×10^{-3} Hz for amplitude and frequency estimations, respectively, and the FMP presented 16.4×10^{-3} pu and 58.7×10^{-3} Hz, for amplitude and frequency, respectively.

5.2.6 Microgrid

The ALMP estimations under frequency deviation, and in the presence harmonics and interharmonics are evaluated considering the signal from a microgrid presented in Eq. (6):

$$\begin{aligned} i_s(t) = & 0.03 \sin(2\pi 24t) + 0.024 \sin(2\pi 48t) + \sin(2\pi 60t) \\ & + 0.023 \sin(2\pi 96t) + 0.029 \sin(2\pi 264t) + 0.03 \sin(2\pi 384t) \\ & + 0.003 \sin(2\pi 588t) + 0.004 \sin(2\pi 708t). \end{aligned}$$

In order to assess the proper estimation of the ALMP algorithm, the ALMP was compared with the MPM and FMP algorithms, since the DFT and HIESSD present spectral leakage for this signal. The signal was simulated considering a noise rate of 40 dB. The errors presented by the algorithm in the estimation of each component in terms of amplitude, frequency and phase are presented in Figs. 23(a), 23(b) and 23(c), respectively.

(a) Mean absolute errors in estimation of amplitude.

(b) Mean absolute errors in the estimation of frequency*.

(c) Mean absolute errors in the estimation of phase*.

Figure 23: Estimation of the frequency components of the microgrid signal added with a white Gaussian noise with SNR=40 dB. The components marked by (*) were not estimated by the MPM algorithm.

It is possible to observe in the Figure 23 that the MPM algorithm is not capable of estimating all the frequency components present in a microgrid signal, the interharmonics of the order 0.4 and 4.6 were not estimated by the algorithm, which resulted in large average errors of 9.2×10^{-4} pu, 49.8 Hz and 47.76° for amplitude, frequency and phase estimations, respectively. Regarding the amplitude estimation, the ALMP presented the best average error of 2.6×10^{-4} pu against 3.4×10^{-4} pu presented by the FMP algorithm. However, considering the frequency and phase estimations the FMP overcame the ALMP algorithm, since it presented average errors of 0.06 Hz and 0.82° for frequency and phase estimations, respectively. The ALMP presented larger average errors in frequency and phase of 0.08 Hz and 2.25° , respectively. However this is compensated by the spectrum analysis, in which the FMP algorithm presents the estimation of several non-existent frequency components, besides the ones that compound the microgrid signal. Figs. 24(a)-(e) show the spectrum resulting from the estimation performed by the DFT, MPM, FMP, HIESSD and ALMP algorithms. It is possible to observe the spectral leakage in the DFT and HIESSD estimations, the absence of two interharmonics components in the MPM estimation and the presence of non-existent components in the FMP estimation. The fundamental frequency component is not entirely presented in the Figures in order to allow a better view of the harmonics and interharmonics components.

(a) Spectrum resulting from DFT algorithm.

(b) Spectrum resulting from MPM algorithm.

(c) Spectrum resulting from HIESSD algorithm with resolution of 2.5 Hz.

(d) Spectrum resulting from FMP algorithm.

(e) Spectrum resulting from ALMP algorithm.

Figure 24: The estimated frequency spectra resulting from the signal decomposition of the microgrid's signal with SNR = 40 dB.

5.2.7 Execution Time

The algorithms ALMP, HIESSD, DFT, MPM, and FMP were implemented using MATLAB software on a PC with Intel Core i7-6700 3.40 GHz Processor. The average execution time was measured for each algorithm to estimate the frequency components for the presented signals, the results are presented in Table 5.

Table 5: Execution time presented by each algorithm considering the signals contaminated with harmonics, interharmonics and frequency deviation.

Algorithm	I		II		III	IV	IV +0.3 Hz	V	VI
	40 dB	40 dB	20 dB	10 dB	$l=5, 40 \text{ dB}$	10 dB	40 dB	40 dB	40 dB
DFT	$4.91 \times 10^{-5} \text{ s}$	$3.69 \times 10^{-5} \text{ s}$	$3.40 \times 10^{-5} \text{ s}$	$2.66 \times 10^{-5} \text{ s}$	$3.29 \times 10^{-4} \text{ s}$	$3.19 \times 10^{-5} \text{ s}$	$3.84 \times 10^{-5} \text{ s}$	$4.19 \times 10^{-5} \text{ s}$	$3.31 \times 10^{-5} \text{ s}$
MPM	1.75 s	1.64 s	1.46 s	1.34 s	1.45 s	1.44 s	1.45 s	1.43 s	1.41 s
FMP	0.42 s	0.44 s	0.48 s	0.44 s	0.40 s	0.44 s	0.39 s	0.47 s	0.41 s
HIESSD	1.45 s	1.28 s	1.11 s	0.77 s	4.72 s	0.59 s	5.87 s	1.35 s	4.61 s
ALMP	0.04 s	0.04 s	0.03 s	0.02 s	0.06 s	0.02 s	0.02 s	0.02 s	0.03 s

The MPM algorithm presented the highest times of execution, followed by HIESSD, FMP, the ALMP and DFT. The ALMP estimated accurately the real values of harmonics and interharmonics, estimating harmonics with results similar to the DFT and HIESSD and estimating interharmonics with results better than the DFT and FMP. In addition, the ALMP algorithm overcame the HIESSD algorithm, since it presented similar accuracy, but with a lower execution time and it provided support for a correct estimation of components in the presence of frequency deviation.

5.3 CHAPTER REVIEW

This chapter presented the results obtained with the ALMP algorithm for the data extraction from signals distorted by harmonics, interharmonics, noise and frequency deviation. Its performance was compared with others well-known algorithms by the calculation of the

MAE and execution times. It presented a performance comparable and even superior from the algorithms in the literature, considering the test conditions presented in this work.

6 FINAL CONSIDERATIONS

This work proposed a novel algorithm for harmonics and interharmonics estimations regarding its robustness and execution time. The ALMP algorithm is a MP-based technique and extracts the components of the signal iteratively until the energy of the residue achieves a low value. It was showed that the algorithm is capable of providing satisfactory estimations considering signal under the presence of Gaussian white noise, frequency deviation and interharmonics presence. A study about the state-of-the-art optimization algorithms for unconstrained problems were presented and the choice of the optimization algorithm was justified through experimental analysis. The Levenberg-Marquadt method overcame the Nelder-Mead and Quasi-Newton methods, due to its processing time up to 1.7 and 1.6 times faster than the NMA and QN methods, respectively and reduced number of floating point operations, about 15k fewer operations than the other two methods.

After the LMA selection, a comparative analysis between the most recent methods for harmonics and interharmonics estimation took place. Regarding the harmonics estimation, the ALMP and DFT algorithms presented lower MAE than MPM, FMP and HIESSD for amplitude and phase estimations. The ALMP algorithm presented the lower MAE for both amplitude and phase estimations, presenting a MAE of 1.6×10^{-3} for amplitude and 0.06° for phase, while the DFT presented 1.8×10^{-3} and 0.11° for amplitude and phase estimations, respectively.

Besides that, the ALMP algorithm estimated the frequency components even under extremely noisy conditions up to 10 dB, and its performance was equivalent to the DFT and FMP algorithms. The MPM and HIESSD presented more susceptibility to noise, the MPM did not estimate all the components and the HIESSD presented larger errors in phase estimations. Regarding the interharmonics estimation, the ALMP algorithm presented the best performance in amplitude estimation among the non-fixed grid algorithms, presenting an average error rate 2.6 times lower than the FMP and 13.5 times lower than the MPM. In addition, the ALMP algorithm presented a good performance under frequency deviation conditions, presenting the lowest error among the algorithms capable of identify frequency deviation. It presented 6.8×10^{-3} pu in amplitude estimation, against 8.1×10^{-3} pu and 16.4×10^{-3} pu presented by the MPM and FMP algorithms.

The ALMP algorithm presented a good performance for all the cases analyzed. It obtained results similar to very well-known and used methods, as DFT and MPM, and overcame many drawbacks of the methods in the literature. It allied the main advantages of each method and achieved a very low processing time, just being slower than the DFT. Regarding the considered criteria and the estimation methods, the ALMP was the only one capable of presenting a good performance in all the cases, i.e. it presented a lower error in harmonics estimation considering noise robustness, unlike the MPM, FMP and HIESSD algorithms; it presented a good performance in interharmonics estimation, unlike the DFT algorithm; it presented robustness to frequency deviation, unlike the DFT and HIESSD algorithms; and presented a reduced execution time, in contrast to the MPM, FMP and HIESSD algorithms.

Besides the results achieved, the main contributions provided by the ALMP algorithm consist in the use of a non-fixed Fourier dictionary, which provides the robustness of the algorithm and the establishment of a non-linear least squares problem based on the energy of the residue which allows the algorithm to deal with fundamental frequency deviations. The proposed algorithm mitigated the failures presented in the HIESSD algorithm, which was the ALMP's predecessor technique, proposed by the same research group. In addition, the ALMP algorithm is suitable for applications with processing limitations, since it is an easy-to-implement algorithm, that is capable of achieving results similar to more complex algorithms in the literature. Regarding future works, it is proposed the development of an embedded real time meter with the ALMP algorithm, for the obtaining and estimation of parameters from real electric power systems.

REFERENCES

- AKANSU, A. N.; HADDAD, R. A. Chapter 6 - wavelet transform. In: AKANSU, A. N.; HADDAD, R. A. (Ed.). **Multiresolution Signal Decomposition (Second Edition)**. Second edition. San Diego: Academic Press, 2001. p. 391 – 442. ISBN 978-0-12-047141-6. Disponível em: <<http://www.sciencedirect.com/science/article/pii/B9780120471416500069>>.
- ALAMO, J. L. del. A comparison among eight different techniques to achieve an optimum estimation of electrical grounding parameters in two-layered earth. **IEEE Transactions on Power Delivery**, v. 8, n. 4, p. 1890–1899, 1993.
- AMBROSE, J. What are the questions raised by the uk's recent blackout? **The Guardian**, ago 2019. Disponível em: <<https://www.theguardian.com/business/2019/aug/12/what-are-the-questions-are-raised-by-the-uks-recent-blackout>>.
- ARRILLAGA, J.; WATSON, N. R. **Power System Harmonics**. Chichester, UK: John Wiley & Sons, Ltd, 2003. 1–15 p. ISBN 0470851295. Disponível em: <<http://doi.wiley.com/10.1002/0470871229>>.
- AYRES, F. J. **Schaum's Outline of Theory and Problems of Matrices**. [S.l.]: Schaum New York, 1962.
- BAUDZUS, L.; KRUMMRICH, P. M. Modified downhill simplex method for fast adaption of optical filters in optical communication systems. **Electronics Letters**, v. 55, n. 8, p. 471–473, 2019.
- BOLLEN, M. H. J.; GU, I. Y.-H. **Signal Processing of Power Quality Disturbances**. Hoboken, NJ, USA: John Wiley & Sons, Inc., 2006. ISBN 9780471931317. Disponível em: <<http://doi.wiley.com/10.1002/0471931314>>.
- CAUDILL, S. B. Estimating the circle closest to a set of points by maximum likelihood using the bhhh algorithm. **European Journal of Operational Research**, v. 172, n. 1, p. 120 – 126, 2006. ISSN 0377-2217. Disponível em: <<http://www.sciencedirect.com/science/article/pii/S0377221704006587>>.
- CHANG, G. W.; CHEN, C. An accurate time-domain procedure for harmonics and interharmonics detection. **IEEE Transactions on Power Delivery**, v. 25, n. 3, p. 1787–1795, 2010.
- CHEN, S. S.; DONOHO, D. L.; SAUNDERS, M. A. Atomic decomposition by basis pursuit. **SIAM Review**, v. 43, n. 1, p. 129–159, 2001. Disponível em: <<https://doi.org/10.1137/S003614450037906X>>.
- CHEN, W. et al. Simplex search-based brain storm optimization. **IEEE Access**, v. 6, p. 75997–76006, 2018.
- CHRYSOCHOS, A. I.; PAPADOPOULOS, T. A.; PAPAGIANNIS, G. K. Robust calculation of frequency-dependent transmission-line transformation matrices using the levenberg–marquardt method. **IEEE Transactions on Power Delivery**, v. 29, n. 4, p. 1621–1629, 2014.

CONN, A.; GOULD, N.; TOINT, P. Convergence of quasi-newton matrices generated by the symmetric rank one update. **Mathematical Programming**, v. 50, p. 177–195, 03 1991.

DUGAN, R. C. et al. **Electrical Power Systems Quality**. [S.l.: s.n.], 2002. ISBN 0-07-138622-X.

ELAD, M. Sparse and redundant representations: From theory to applications in signal and image processing. **Sparse and Redundant Representations: From Theory to Applications in Signal and Image Processing**, p. 1–376, 2010. ISSN 144197010X.

GRIVA, I.; NASH, S. G.; SOFER, A. Appendix d: Nonlinear least squares for data fitting. 1996. Disponível em: <<http://math.gmu.edu/~igriva/book/Appendix%20D.pdf>>.

HUA, Y.; SARKAR, T. K. Matrix pencil method and its performance. In: **ICASSP-88., International Conference on Acoustics, Speech, and Signal Processing**. [S.l.: s.n.], 1988. p. 2476–2479 vol.4.

IEC. **IEC 61000-4-7 - Electromagnetic compatibility (EMC) – Part 4-7: Testing and measurement techniques – General guide on harmonics and interharmonics measurements and instrumentation, for power supply systems and equipment connected thereto**. [S.l.], 2006.

IEEE. Ieee recommended practice and requirements for harmonic control in electric power systems. **IEEE Std 519-2014 (Revision of IEEE Std 519-1992)**, p. 1–29, June 2014.

IKRAMOV, K. Matrix pencils: Theory, applications, and numerical methods. **Journal of Soviet Mathematics**, v. 64, n. 2, p. 783–853, 1993. ISSN 1573-8795.

IOZSA, A.; VESA, A. The esprit algorithm. variants and precision. In: **2010 9th International Symposium on Electronics and Telecommunications**. [S.l.: s.n.], 2010. p. 165–168.

JIN, Z. et al. A robust and adaptive detection scheme for interharmonics in active distribution network. **IEEE Transactions on Power Delivery**, v. 33, n. 5, p. 2524–2534, 2018.

KUMAR, R. et al. Dual-Tree Complex Wavelet Transform-Based Control Algorithm for Power Quality Improvement in a Distribution System. **IEEE Transactions on Industrial Electronics**, v. 64, n. 1, p. 764–772, jan 2017. ISSN 0278-0046. Disponível em: <<http://ieeexplore.ieee.org/document/7464916/>>.

LEI, C. et al. Method based on sparse signal decomposition for harmonic and inter-harmonic analysis of power system. **Journal of Electrical Engineering and Technology**, v. 12, n. 2, p. 559–568, 03 2017.

LI, J.; LIU, H. A contracting bfgs update in quasi-newton methods for unconstrained optimization. In: **2019 15th International Conference on Computational Intelligence and Security (CIS)**. [S.l.: s.n.], 2019. p. 59–62.

LOBOS, T.; LEONOWICZ, Z.; REZMER, J. Harmonics and interharmonics estimation using advanced signal processing methods. **Ninth International Conference on Harmonics and Quality of Power**, v. 1, p. 335–340, 2000. Disponível em: <<http://ieeexplore.ieee.org/lpdocs/epic03/wrapper.htm?arnumber=897050>>.

LOURAKIS, M. A brief description of the levenberg-marquardt algorithm implemented by lev-mar. **A Brief Description of the Levenberg-Marquardt Algorithm Implemented by Levmar**, v. 4, 01 2005.

MALLAT, S. G.; ZHANG, Z. Matching pursuits with time-frequency dictionaries. **IEEE Transactions on Signal Processing**, v. 41, n. 12, p. 3397–3415, 1993.

MARZ, M. B. Interharmonics : What They Are , Where They Come From and What They Do. In: **Minnesota Power Systems Conference Papers (MIPSYCON)**. [S.l.: s.n.], 2016. p. 1–8.

MATHEWS, J.; FINK, K. **Numerical Methods Using Matlab**. New Jersey, USA: Pretience-Hall, 2004.

MCKINNON, K. Convergence of the nelder-mead simplex method to a non-stationary point. 09 1996.

NELDER, J. A.; MEAD, R. A Simplex Method for Function Minimization. **The Computer Journal**, v. 7, n. 4, p. 308–313, 01 1965. ISSN 0010-4620. Disponível em: <<https://doi.org/10.1093/comjnl/7.4.308>>.

NOCEDAL, J.; WRIGHT, S. **Numerical Optimization**. Springer New York, 2006. Disponível em: <<https://doi.org/10.1007%2F978-0-387-40065-5>>.

OPPENHEIM, A. V. **Discrete-time signal processing**. [S.l.]: Pearson Education India, 1999.

ORTBANDT, C. et al. Parameter estimation in electrical power systems using prony's method. **Journal of Physics: Conference Series**, IOP Publishing, v. 659, p. 012013, nov 2015. Disponível em: <<https://doi.org/10.1088%2F1742-6596%2F659%2F1%2F012013>>.

PRADO, T. D. A.; BARROS, A. M.; GUARNERI, G. A. An algorithm based on sparse decomposition for estimating harmonic and interharmonic components of stationary signals in power systems. **IEEE Access**, v. 7, p. 163958–163968, 2019.

PRAMANIK, M.; MITRA, P. Parameter estimation of multi-transient signal using band-pass matrix pencil method. In: **2017 7th International Conference on Power Systems (ICPS)**. [S.l.: s.n.], 2017. p. 436–440.

QIAN, H. Counting the floating point operations (flops). **MATLAB Central File Exchange**, 2020. Disponível em: <<https://www.mathworks.com/matlabcentral/fileexchange/50608-counting-the-floating-point-operations-flops>>.

QIAO, J. et al. Adaptive levenberg-marquardt algorithm based echo state network for chaotic time series prediction. **IEEE Access**, v. 6, p. 10720–10732, 2018.

RANGANATHAN, A. The levenberg-marquardt algorithm. **Academia.edu**, 07 2004.

RAVINDRAN, V.; RÖNNBERG, S. K.; BOLLEN, M. H. J. Interharmonics in pv systems: a review of analysis and estimation methods; considerations for selection of an apt method. **IET Renewable Power Generation**, v. 13, n. 12, p. 2023–2032, 2019.

RICE, D. E. A Detailed Analysis of Six-Pulse Converter Harmonic Currents. **IEEE Transactions on Industry Applications**, v. 30, n. 2, p. 294–304, 1994.

- RODRÍGUEZ, A. F. et al. Coding Prony's method in MATLAB and applying it to biomedical signal filtering. **BMC Bioinformatics**, v. 19, p. 451, 2018.
- RUBINSTEIN, R.; BRUCKSTEIN, A. M.; ELAD, M. Dictionaries for Sparse Representation Modeling. **Proceedings of the IEEE**, v. 98, n. 6, p. 1045–1057, jun 2010. ISSN 0018-9219. Disponível em: <<http://ieeexplore.ieee.org/document/5452966/>>.
- SALMAN, M. S.; KUKRER, O.; HOCANIN, A. A fast quasi-newton adaptive algorithm based on approximate inversion of the autocorrelation matrix. **IEEE Access**, v. 8, p. 47877–47887, 2020.
- SCHÄFER, B. et al. Non-gaussian power grid frequency fluctuations characterized by lévy-stable laws and superstatistics. **Nat Energy**, v. 3, p. 119–126, 2018.
- SHESHYEKANI, K. et al. A General Noise-Resilient Technique Based on the Matrix Pencil Method for the Assessment of Harmonics and Interharmonics in Power Systems. **IEEE Transactions on Power Delivery**, v. 32, n. 5, p. 2179–2188, 2017. ISSN 08858977.
- SINGER, S.; NELDER, J. Nelder-Mead algorithm. **Scholarpedia**, v. 4, n. 7, p. 2928, 2009. revision #91557.
- SPYROPOULOS, D.; MITRONIKAS, E. A Review on the Faults of Electric Machines Used in Electric Ships. **Advances in Power Electronics**, v. 2013, 2013.
- SUN, J. et al. Robust frequency, phase, and amplitude estimation in power systems considering harmonics. **IEEE Transactions on Power Delivery**, v. 35, n. 3, p. 1158–1168, 2020.
- SYAM, P. et al. Simulation and experimental study of interharmonic performance of a cycloconverter-fed synchronous motor drive. **IEEE Transactions on Energy Conversion**, v. 19, n. 2, p. 325–332, 2004.
- TESTA, A. et al. Interharmonics: Theory and modeling. **IEEE Transactions on Power Delivery**, v. 22, n. 4, p. 2335–2348, 2007. ISSN 08858977.
- TROPP, B. J. a.; WRIGHT, S. J.; TROPP, J. a. Computational Methods for Sparse Solution of Linear Inverse Problems. **Proceedings of the IEEE**, v. 98, n. 6, p. 948–958, 2010. ISSN 0018-9219.
- WALLACE, A. K.; WARD, E. S.; WRIGHT, A. Sources of harmonic currents in slip-ring induction motors. **Proceedings of the Institution of Electrical Engineers**, v. 121, n. 12, p. 1495–1500, December 1974. ISSN 0020-3270.
- WANG, J.; YANG, F.; DU, X. Microgrid harmonic and interharmonic analysis algorithm based on cubic spline interpolation signal reconstruction. In: **IEEE PES Innovative Smart Grid Technologies**. [S.l.: s.n.], 2012. p. 1–5.
- YANG, J.; ZOU, J. Parameter estimation of a horizontally multilayered soil with a fast evaluation of the apparent resistivity and its derivatives. **IEEE Access**, v. 8, p. 52652–52662, 2020.
- ZHAO, C. et al. Novel method based on variational mode decomposition and a random discriminative projection extreme learning machine for multiple power quality disturbance recognition. **IEEE Transactions on Industrial Informatics**, v. 15, n. 5, p. 2915–2926, 2019.

ZHENG, W.; WU, W. An adaptive distributed quasi-newton method for power system state estimation. **IEEE Transactions on Smart Grid**, v. 10, n. 5, p. 5114–5124, 2019.

ZHOU, W.; ZHANG, L. Global convergence of a regularized factorized quasi-Newton method for nonlinear least squares problems. **Computational & Applied Mathematics**, scielo, v. 29, p. 195 – 214, 06 2010. ISSN 1807-0302. Disponível em: <http://www.scielo.br/scielo.php?script=sci_arttext&pid=S1807-03022010000200006&nrm=iso>.

ZYGARLICKI, J.; MROCZKA, J. Prony's method used for testing harmonics and interharmonics in electrical power systems. **Metrology and Measurement Systems**, v. 19, p. 659–672, 01 2012.