UNIVERSIDADE TECNOLÓGICA FEDERAL DO PARANÁ

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BALANCING AND SEQUENCING OF AN AUTOMOTIVE ASSEMBLY LINE THROUGH MIXED INTEGER LINEAR PROGRAMMING

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Balanceamento e Sequenciamento de uma Linha de Montagem Automotiva por meio de Programação Linear Inteira Mista

Dissertação apresentada como requisito para obtenção do grau de Mestre em Ciências - Área de concentração em Engenharia de Automação e Sistemas do Programa de Pós-Graduação em Engenharia Elétrica e Informática Industrial da Universidade Tecnológica Federal do Paraná.

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Trabalho de pesquisa de mestrado apresentado como requisito para obtenção do título de Mestre Em Ciências da Universidade Tecnológica Federal do Paraná (UTFPR). Área de concentração: Engenharia De Automação E Sistemas.

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"The more you know, the more you realize you
don’t know"
Aristotle.

RESUMO

As empresas buscam constantemente maneiras de melhorar a sua produtividade e lucro. Além disso, linhas de montagem devem ser flexíveis e produzir diferentes produtos na mesma linha de produção para serem competitivas no mercado atual, no qual a customização está em alta. O balanceamento de linha é uma ferramenta que pode aumentar a produção e diminuir o tempo ocioso dos operadores e estações de trabalho. Como o balanceamento requer mudar a alocação de tarefas entre estações é, em termos práticos, considerada uma decisão de médio ou longo prazo. De modo complementar, o sequenciamento da produção é considerado uma decisão de curto prazo, é normalmente aplicado para uma dada distribuição de tarefas conhecida nas estações, e define a ordem na qual os modelos de produto serão produzidos para um horizonte de análise relativamente curto (algumas horas ou dias, a depender da linha produtiva). Este estudo tem como objetivo balancear uma linha de montagem real de veículos. Também busca-se balancear e sequenciar um problema teórico inspirado pela linha real. Propõem-se o uso de Programação Linear Inteira Mista para tecnicamente auxiliar as tarefas de balanceamento e sequenciamento. O trabalho inicialmente revisa conceitos bibliográficos relevantes ao tema. Em seguida o problema real é descrito, mostrando os diferentes modelos produzidos na linha, seus conteúdos de trabalho e o diagrama de precedência entre tarefas. Uma metodologia de resolução foi proposta com três diferentes análises, cada qual com seu modelo matemático, as quais foram elaboradas na medida em que o aprofundamento nas características do problema ocorria. A Análise 1 buscou encontrar o balanceamento de tarefas para oito cenários produtivos distintos (*mix* produtivos) observados durante seis meses de produção. A referida análise encontrou uma solução com melhora de 10.1% no tempo de ciclo, mas falhou em atender a demanda de um *mix* específico. Na sequência, a Análise 2 visou balancear cada *mix* específico, depois adicionou uma variável para limitar o número de mudanças no balanceamento e encontrar uma solução resiliente a mudanças na demanda, obtendo 12.0% de redução no tempo ciclo. Essa análise foi capaz de atender todos os oito *mix* de produção presentes no período de seis meses, foi validada pelo especialista de linha e implementada. Finalmente, a Análise 3 explorou a possibilidade de se realizar o balanceamento e o sequenciamento de modo simultâneo em um problema teórico inspirado no problema em análise. Novamente buscou-se encontrar uma única solução capaz de atender diferentes demandas (*mix* produtivos), com uma redução de 17.1% no tempo de ciclo de estado transiente, em relação ao estado inicial da linha. A Análise 3 evidencia que a realização do balanceamento e sequenciamento em uma abordagem integrada tende a propiciar resultados mais aderentes ao potencial produtivo da linha considerada.

Palavras-chave: balanceamento de linha de montagem; sequenciamento de linha de montagem; programação linear inteira mista; linha de montagem automotiva.

ABSTRACT

Companies are constantly looking for ways to improve their productivity and profits. Furthermore, assembly lines must be flexible and produce different products on the same production line to compete in the current market, in which customization is rising. Line balancing is a tool that can increase production and reduce idle time for operators and workstations. As balancing requires changing the allocation of tasks between stations, it is, in practical terms, considered a medium or long-term decision. In a complementary way, production sequencing is considered a short-term decision; it is usually applied to a given known distribution of tasks across stations, and defines the order in which product models will be produced for a relatively short analysis horizon (some hours or days, depending on the production line). This study aims to balance a real vehicle assembly line in the automotive industry. In addition, balance and sequence a theoretical problem inspired by the real line. Mixed Integer Linear Programming is proposed to technically assist in balancing and sequencing tasks. The work initially reviews bibliographic concepts relevant to the topic. Then, the real problem is described, showing the different models produced on the line, their work contents, and the precedence diagram between tasks. A resolution methodology was proposed with three different analyses, each with its mathematical model. These analyses were developed as the problem's characteristics were deepened. Analysis 1 sought to find the balance of tasks for eight different production scenarios (productive mix) observed during six months of practical production. Analysis 1 found a solution with a 10.1% improvement in cycle time but failed to meet the demand of a specific mix. Next, Analysis 2 aimed to balance each specific mix, then added a variable to limit the number of changes in balance and find a solution resilient to changes in demand, obtaining a 12.0% reduction in cycle time. This analysis met all eight productive mixes present in the six-month period, was validated by the line specialist, and was implemented. Finally, Analysis 3 explored the possibility of performing balancing and sequencing simultaneously in a theoretical problem inspired by the considered problem. Once again, it is sought to find a single solution capable of meeting different demands (productive mix), with a 17.1% reduction in the steady-state cycle time in relation to the initial state of the line. Analysis 3 shows that carrying out balancing and sequencing in an integrated approach tends to provide results that are more aligned with the productive potential of the line considered.

Keywords: assembly line balancing; assembly line sequencing; mixed integer linear programming; automotive assembly line.

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LIST OF ABBREVIATIONS AND ACRONYMS

ALB	Assembly Line Balancing
AON	Activity-On-Node
BD	Balancing Delay
BIW	Body-In-White
CT	Cycle Time
CTST	Steady-State Cycle Time
ERALBP	Economically Robust Assembly Line Balancing Problem
HSI	Horizontal Smoothness Index
IT	Idle Time
LE	Line Efficiency
MALBP	Mixed-Model Assembly Line Balancing Problem
MILP	Mixed Integer Linear Programming
MMAL	Mixed-Model Assembly Line
MPS	Minimum Part Set
OR	Operations Research
SALBP	Simple Assembly Line Balancing Problem
SI	Smoothness Index

SUMMARY

1 INTRODUCTION

Within the current production context, fabricating companies tend to embrace a production flow with assembly lines that share different products on the same line (BECKER; SCHOLL, 2006). In this context, balancing the workload of operators and stations is a vital activity known as line balancing. Line balancing allows the reduction of the difference of times between workstations and operators involved, minimizing the idleness present in the system and increasing the output of products (BATTAÏA; DOLGUI, 2013). To minimize the effects of having products with different workloads, sequencing the production allows for better use of the assembly line with short-term decisions, with detailed planning of sequences to produce (BOYSEN; FLIEDNER; SCHOLL, 2009b).

This dissertation uses mathematical programming to bring the balancing of a real-world assembly line in the automotive industry. It also performs studies of the balancing and sequencing of a theoretical problem inspired by the real line facing difficulties in meeting customer demand. The initial considerations about the subject are given in Section 1.1, the general objective in Subsection 1.2.1, and specific objectives in Subsection 1.2.2. Section 1.3 brings the justification of the study, and finally, Section 1.4 shows the study structure.

1.1 Initial considerations

Assembly lines were first designed to mass produce the Ford Model T, and the classical phrase of Henry Ford resembles the production philosophy at that time: “Any customer can have a car painted in any color that he wants so long as it is black” (FORD, 1922). In last decades, the automotive industry is working according to society’s customization needs, with such a great catalog of options that theoretically results in, for instance, more than a thousand models of the manufacturer BMW even in the beginnings of 2000’s (MEYR, 2004). This context indicates that the flexibility of the assembly lines is more valuable, but without losing their efficiency, so companies are always seeking to minimize waste and maximize output in order to stay competitive.

Besides this factor, extraordinary events such as the shortage of raw material, the lack of labor (caused, for example, by health crises), and contracts to supply a large number of goods lead to a production oscillation concerning the customization demand and production capacity, which shows even more the importance of flexibility of the production lines.

To minimize these effects, line balancing has been increasingly used to minimize the idle time of production lines and increase their efficiency (BATTAÏA; DOLGUI, 2013). As a rule, solution approaches based on mathematical models are used to find an optimized solution for a balancing problem: the workload distribution between stations and workers on the line under analysis.

Another factor that influences the line efficacy is related to the sequencing of products in the considered production line. Dedicated lines do not require any sequencing of products that flow in the line because they only produce one model of a product. Conversely, mixed-model assembly lines, which produce more than one model of a product in the same line in an intermixed sequence, are hugely influenced by the workloads of different products that share the same line (BOYSEN; FLIEDNER; SCHOLL, 2009b). Balancing and sequencing aspects are involved in the productive optimization of the considered study.

1.2 Objectives

This section presents the general and specific objectives of the study.

1.2.1 General objective

To develop a solution approach based on Mixed Integer Linear Programming (MILP) to balance the production of a real-world automotive assembly line, aiming to increase the line productivity, while being flexible to try to respect different customer demands. In addition, to study the balancing and sequencing of a theoretical line inspired by the real one.

1.2.2 Specific objectives

To achieve the general objective, the following steps were required:

- To compile all the tasks performed on the assembly line for all different model variants (product models) and to define the constraints and the precedences between the tasks.
- To create a MILP model to balance the line and validate its feasibility.
- To generate an optimized operational proposal, analyze feasibility, implement changes in the real line, and compare the results obtained for the practical application through line-balancing indicators.
- To create a MILP model to balance and sequence a theoretical line inspired by the real one.

1.3 Justification of the study

To remain competitive, industries must find ways to be always more efficient. That could be done by reducing the shop stock of parts (MARQUEZE; KOVALESKI; MAGATÃO, 2022) or through a line balancing to produce more parts with fewer operators (MARQUEZE; MEIRA; MAGATÃO, 2022).

However, being efficient becomes more complex when the company has to produce different options to satisfy the customers. Car manufacturers can offer different models of cars, such as compacts, sports cars, SUVs, luxury cars, pickups, and minivans. In addition, each car model can have different customizations, leading to even more options to create an almost unique car.

In order to be able to mass produce while being flexible, mixed-model lines are used, but the difference in work content between the different products can lead to a significant difference in workload between operators, which can cause an increase in the idle times of the line and reduce the total output of parts.

Line balancing through MILP can help improve this issue by finding a solution that can even the distribution of tasks between the stations. However, changing this distribution can be challenging due to the cost of machinery, layout restrictions, and training time, so balancing is considered a medium/long-term solution.

Sequencing the production, however, is considered a short-term solution because it will not change the allocation of tasks. Performing balancing and sequencing simultaneously allows the solution improvement for the mixed-model context and a more complete answer in short-term (sequencing) and medium/long-term (balancing) solutions.

Within the context of industrial optimization mentioned, this study aims to, firstly, balance an automotive assembly line of a multinational company exposed to different manufacturing demands, contributing to making the line under analysis more flexible to manage changes in demanding conditions. The study is based on a utility vehicle production line and carried out *in loco* analyses of the feasibility of the proposed balancing solutions. Afterwards, a theoretical study involving balancing and sequencing was also developed. Practical *in loco* validations were not possible for this last study. However, it allowed to evidence the influence of both elements (balancing and sequencing) on the obtained computational solution, which contributed to indicating that, if possible, both aspects should be taken for a more accurate operational answer.

1.4 Study structure

The remaining of this study is structured as follows. Chapter 2 performs the review of fundamentals and related works. Chapter 3 describes the practical problem. Chapter 4 shares the elaborated mathematical model, highlighting the methodology in which the model is immersed. Chapter 5 shows the results found, and finally, Chapter 6 brings the final considerations and suggestions for continuing the work.

2 CONTEXT AND RELATED WORKS

This chapter brings in Section 2.1 the main concepts of assembly lines. Then, Section 2.2 discusses mathematical programming for line balancing. Section 2.3 presents indicators for line balancing. Finally, Section 2.4 discusses the importance of sequencing in mixed-model assembly lines.

2.1 Assembly line and its classification

An assembly line is a production system where the stations are arranged in a serial manner, product-oriented, and the products generally move through a mechanism, such as a conveyor belt (BOYSEN; FLIEDNER; SCHOLL, 2007). At first, the assembly lines were designed to mass produce only one product, which was the case of the Ford Model T. However, in the last decades, assembly lines need to be able to produce different products while maintaining high efficiency (TSENG; JIAO, 2001; BOYSEN; SCHULZE; SCHOLL, 2022). This context reflects a Mixed-Model Assembly Line (MMAL) condition, where the sharing of a line with different models highlights the need for a manufacturing context flexible to the different characteristics of the products. Saif *et al.* (2014) classify the assembly lines based on different characteristics: layout, workflow, product, task time, and objective.

First, regarding the layout, assembly lines can be serial lines, parallel lines, U-shaped lines, and two-sided assembly lines. Serial lines have stations in a serial disposition, with the product being moved from the first to the last station. Here, the cycle time is defined by the station with the highest workload, and it should also include the dead time, which means the time to transport the parts from one station to the next. Parallel lines divide the workload among parallel stations, which is functional when dealing with large workloads so that it will decrease the cycle time. In U-shaped lines, the product can pass through the station more than once, optimizing the efficiency, decreasing the idle time, and reducing the need to duplicate the stations. The two-sided lines allow working on both sides of the line, with more than one worker or machine working simultaneously (SAIF *et al.*, 2014).

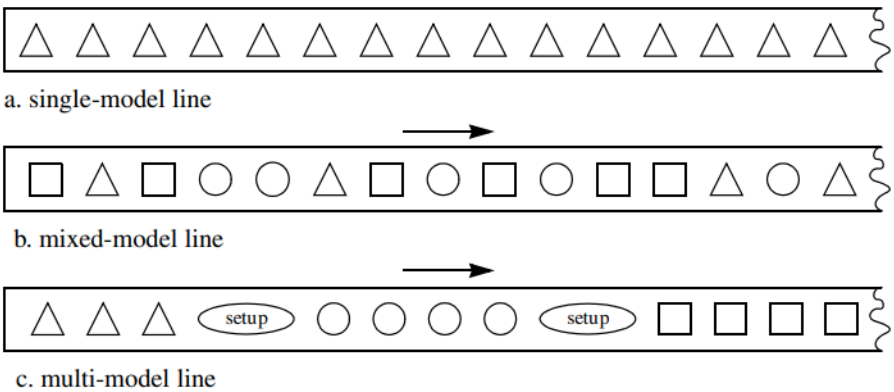
The workflow characteristics are divided into two: paced and unpaced lines. In paced assembly lines, the time to complete the tasks is limited, and such time should be similar to all stations. Unpaced assembly lines are divided into synchronous and asynchronous lines. The first one transfers finished parts from all stations simultaneously after a fixed time. In the latter, each station can have different cycle times, and the operator or machine starts to work as soon as the part (product) becomes available, which can lead to starvation, that is, the previous station has not finished the product, or blockage when there is no room to put the part produced in the station until the next station becomes available. To reduce idle time due to starvation or blockage, buffers are used so that they can temporarily store the parts between the stations (SAIF *et al.*, 2014).

The assembly line balancing problem can have only one objective, such as minimizing cycle time, minimizing the number of stations, minimizing the smoothness index[1], and minimizing design costs. Alternatively, the balancing problem could have multiple objectives, which in most cases will have a trade-off between the different goals (SAIF *et al.*, 2014).

The task time can be separated into three groups: fixed, varying, and stochastic task time. Fixed task times are when the variation of task execution is slight, common in highly reliable machines, where the time is considered deterministic. For manual labor, the task time is generally not fixed; it could vary with the learning of the employee, that is, the first time the worker performs the task, the worker will have one time, which can decrease with the experience acquired. It could also vary with workers' fatigue, machine breakdown, poorly maintained equipment, and defects in the raw material. When the task time is stochastic, the balancing problem becomes more complex, and the time can be represented by a stochastic variable, a fuzzy variable, or an independent normal distributed variable (SAIF *et al.*, 2014).

The assembly lines can be divided into three categories regarding their products: single-model, which produces only one kind (model) of a product; mixed-model, which produces more than one model in an intermixed sequence; and multi-model, which produces a sequence of batches with a setup between them (BECKER; SCHOLL, 2006). Figure 1 exemplifies the different mentioned categories.

Figure 1 – Single, mixed, and multi-model line



Source: (BECKER; SCHOLL, 2006).

When a factory produces several products, one alternative is to have dedicated lines for each product. However, this condition is economically viable only for high volumes of a specific product, which is not the tendency (ZHANG; MING; BAO, 2022). Then an assembly line with multiple models, MMAL, emerges as an alternative. Some advantages of the MMAL are already cited in the early paper of Rao (1971): it provides a continuous flow of each model, reduces finished goods inventory, eliminates the changeover of tools in the assembly line, and offers greater flexibility in production.

[1] This index is afterward exploited in Section 2.3. In essence, it measures the difference between the workload of each considered station.

Flexibility is a vital characteristic in the automotive industry due to many customization options, exploring ways to better meet customer demand (ALFORD; SACKETT; NELDER, 2000), and reach mass customization (PINE, 1993; ZHANG; MING; BAO, 2022). For instance, Pil & Holweg (2004) bring the total number of variations of some vehicles produced in 2002, considering the categories of Body-In-White (BIW), power train, paint-and-trim combinations, and factory-fitted options, available in Table 1. As indicated in this table, the total number of variations can be high (*e.g.*, hundreds of thousands), indicating that decisions on mixed-model lines involve treating combinatorial aspects of the problem.

Table 1 – Number of different models

Model	Bodies	Power trains	Paint-and-trim combinations	Factory-fitted options	Total number of variations
Nissan Micra	2	6	30	4	676
Peugeot 206	3	8	70	5	1739
Nissan Almera	3	5	30	5	3036
Toyota Yaris	2	6	30	8	34320
Fiat Punto	2	5	51	8	39364
Peugeot 307	4	8	70	9	41590
Renault Clio	2	10	57	9	81588
Toyota Corolla	4	5	24	6	162752
Ford Fiesta	2	5	57	13	$1.19 * 10^6$
Renault Megane	2	6	52	14	$3.45 * 10^6$
GM Astra	4	11	83	14	$2.71 * 10^7$
GM Corsa	2	9	77	17	$3.67 * 10^7$
Ford Focus	4	11	64	19	$3.67 * 10^8$
VW Golf	3	16	221	26	$2.00 * 10^9$
Fiat Stilo	3	7	93	25	$1.09 * 10^{10}$
VW Polo	2	9	195	27	$5.26 * 10^{10}$
Mini (BMW)	1	5	418	44	$5.10 * 10^{16}$
BMW 3-Series	3	18	280	45	$6.41 * 10^{16}$
Mercedes C-Class	2	16	312	59	$1.13 * 10^{21}$
Mercedes E-Class	2	15	285	70	$3.35 * 10^{24}$

Source: Adapted from (PIL; HOLWEG, 2004).

2.1.1 Terminology

In order to facilitate the understanding of this work, some key terms used are hereafter described:

- Task - smallest units of work that can be performed interdependently (KRAJEWSKI; RITZMAN; MALHOTRA, 2010);
- Task Duration - time required for an operator or machine to complete the task;
- Station - the location on the assembly line where an operator or machine performs a set of tasks (RAO, 1971);
- Takt Time - demand rate of production. It is calculated by the available time to produce divided by the customer demand (ROTARU, 2008);

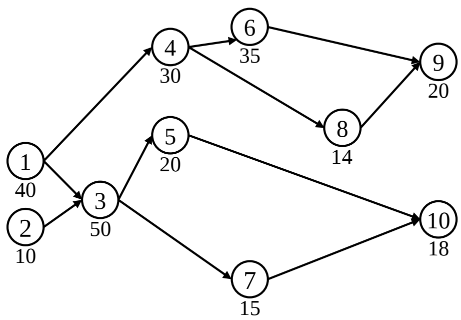
- Cycle Time (CT) - the interval between the output of two consecutive products. The cycle time of the assembly line is the longest of all stations, which means the bottleneck time. If the cycle time is higher than the takt time, the company is failing to deliver to the customer (ROTARU, 2008);

- Workstation Time - the sum of time of all tasks performed on the workstation (RAO, 1971);

- Station Idle Time - the amount of time the operator or machine is idle due to the difference between the cycle time and the workstation time (RAO, 1971).

Additionally, a further key concept is the precedence diagram, which is a diagram that presents the precedence relationship between tasks, that is, tasks that must be performed before others in a defined sequence. It is represented by the Activity-On-Node (AON) network, where the tasks are represented by nodes, with the time to perform them below each node, and the precedence relations are indicated through arrows (KRAJEWSKI, 2009). Figure 2 shows an example of a precedence diagram with ten tasks.

Figure 2 – Example of precedence diagram



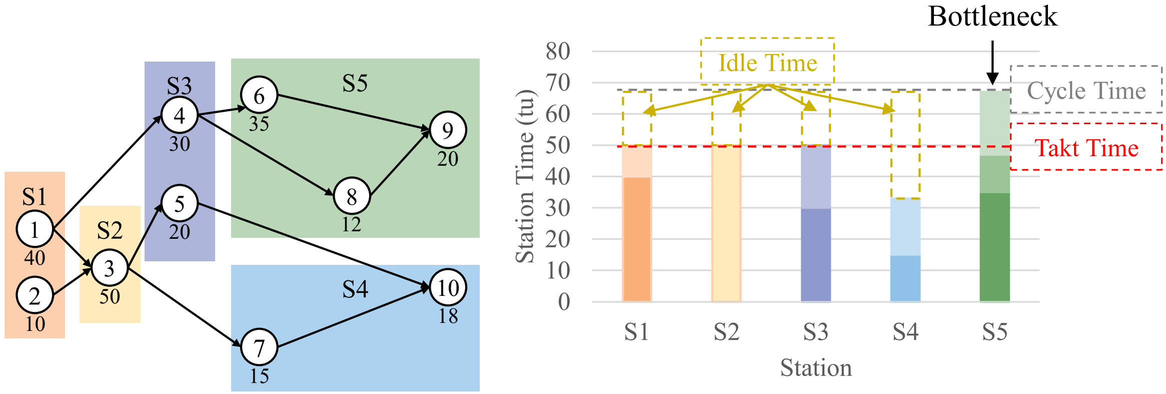
Source: Adapted from (KRAJEWSKI, 2009).

Figure 3 brings an example of a precedence diagram and line balancing. Considering a takt time of 50 time units and five stations, one possible balancing is shown in Figure 3, allocating tasks 1 and 2 on Station 1 (S1), task 3 on S2, tasks 4 and 5 on S3, tasks 7 and 10 on S4 and finally tasks 6, 8 and 9 on S5. On the left, the precedence diagram is shown with the stations, and on the right, a graph with the stations' time exemplifying the concepts of cycle time, takt time, idle time, and bottleneck.

It is possible to notice that Station 5 is the bottleneck, with a cycle time of 67 tu, which is higher than the takt time, meaning that the production cannot satisfy the demand. Since the cycle time is higher than all the other workstation times, the remaining four stations have an idle time. With this in mind, rebalancing the tasks and relocating task 6 to S4 and task 10 to S5, a new solution is found in Figure 4.

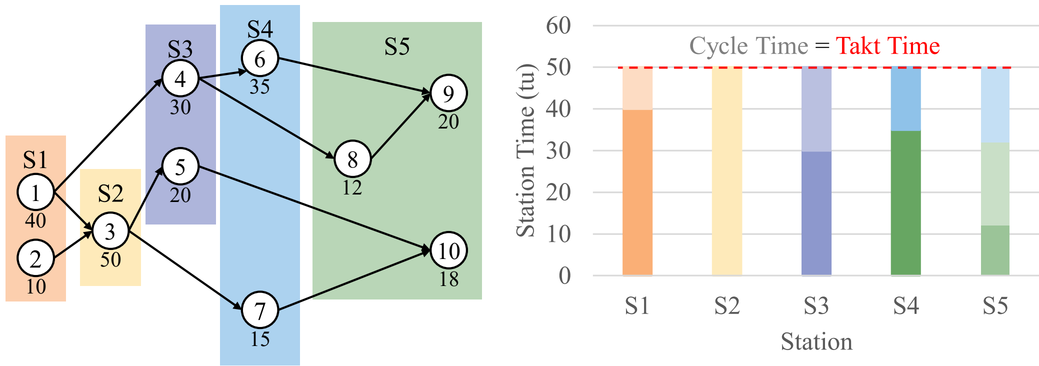
For the (re)balanced condition presented in Figure 4, all five stations have the same cycle time that respects the takt time; in other words, there is no idle time, and the production meets the demand.

Figure 3 – Balancing of task allocation on stations and time distribution between stations



Source: Own authorship (2023).

Figure 4 – New balancing of task allocation on stations and time distribution between stations



Source: Own authorship (2023).

The workstations can also have zone constraints or zoning restrictions. For instance:

- A group of tasks that must be performed in the same station (or in some specific stations); or, on the other hand, that certain tasks could not be allocated together in the same station;
- Tasks that can only be done at one specific station, usually due to machinery constraints or operator skill (Positional constraints);
- Tasks that need an interval between the execution of two tasks, which may be a “distance” in time or in stations (Distance constraints).

The proposed balancing solution has to consider the practical conditions and can be done empirically or with the aid of an analytical tool such as mathematical programming, which is described in Section 2.2.

2.2 Mathematical programming

Mathematical programming is part of the Operations Research (OR) that emerged in World War II when a group of English scientists aimed to find the best allocation of limited military resources. With the war's end, OR began to be used in other fields with complex problems. The striking point is that this tool enables the analysis and decision process based on the use of models, making it possible to test a proposal before implementation. Therefore, it is an important tool in the decision-making process (LISBOA, 2002).

2.2.1 Mathematical programming for line balancing

Specifically for the line balancing, the first mathematical formalization of Assembly Line Balancing (ALB) was made by Salveson (1955). The author formalized that it consists of allocating tasks into stations, respecting precedence constraints so that the station time is the sum of all allocated tasks. To find a feasible line balancing, this time should not exceed the cycle time when provided one (BECKER; SCHOLL, 2006).

The Simple Assembly Line Balancing Problem (SALBP) has some simplifications as follow (BECKER; SCHOLL, 2006):

- mass-production of one homogeneous product;
- paced line with fixed cycle time;
- deterministic operation times;
- no assignments restrictions besides the precedence constraints;
- serial line with one-sided stations;
- all stations are equally equipped regarding machines and workers.

There can be different objectives for SALBP. For instance, SALBP-1 aims to minimize the number of stations S given the cycle time CT; SALBP-2 minimize the cycle time CT given the number of stations S; when both the cycle time and the number of stations can be altered, the SALBP-E is used to maximize the line efficiency LE; finally, SALBP-F seeks a feasible solution given the number of stations S and the cycle time CT (BOYSEN; FLIEDNER; SCHOLL, 2008).

However, the SALBP simplifications make them hard to be used in real assembly line problems. To solve more realistic problems, new models were created, taking into account specific aspects of real problems (BOYSEN; FLIEDNER; SCHOLL, 2007). Different characteristics can be incorporated into the model, such as those mentioned in Section 2.1. Furthermore, assignment restrictions can be included in the SALBP. For instance: distance between tasks, meaning that they have to be performed with an exact, minimum or maximum distance (in stations S);

incompatibility between tasks, which makes it impossible to allocate them in the same station; fixed tasks, which can only be performed in a specific station. Specific mathematical programming models have been developed to model and solve SALBP and extensions as indicated by Scholl (1999). For example, the Economically Robust Assembly Line Balancing Problem (ERALBP) (LOPES *et al.*, 2021) expands the SALBP to meet different production demands, finding an optimal solution for the project of a production line with the condition of demand variability throughout time. Its biggest goal is to avoid the need to perform a new balancing at each demand variation. Specific mathematical programming formulations are used by the authors. Some other works go further and address the uncertainty in demand, product sequence, and buffer allocation (SIKORA, 2022).

Not only SALBP (and variants) has been addressed by mathematical programming, but also the Mixed-Model Assembly Line Balancing Problem (MALBP). MALBP considers that the assembly line produces more than one product in the same line without setups in between. The task times could be averaged concerning the estimated demand of respective models in the model mix in order to simplify the model and work as a SALBP (BOYSEN; FLIEDNER; SCHOLL, 2009a) or could have one time for each task according to each model, what increases the complexity of the mathematical model. Likewise to the SALBP, the MALBP can be categorized into MALBP-1, MALBP-2, MALBP-E, and MALBP-F (SCHOLL, 1999). Thus, mathematical programming-based approaches are used to model and solve production line balancing problems (BOYSEN; SCHULZE; SCHOLL, 2022).

2.3 Line balancing: Objective functions and indicators

The two most common objective functions in the line balancing literature are (BATTAÏA; DOLGUI, 2013): the minimization of the total idle time for a given cycle time (Type I balancing), which is also equivalent to the minimization of the number of stations for a given cycle time; or, cycle time minimization for a given number of stations (Type II balancing). These two objectives can be used for SALBP or MALBP operational conditions.

However, it is possible to find different objective functions, as listed by (SUGUINOSHITA; MAGATÃO, 2018):

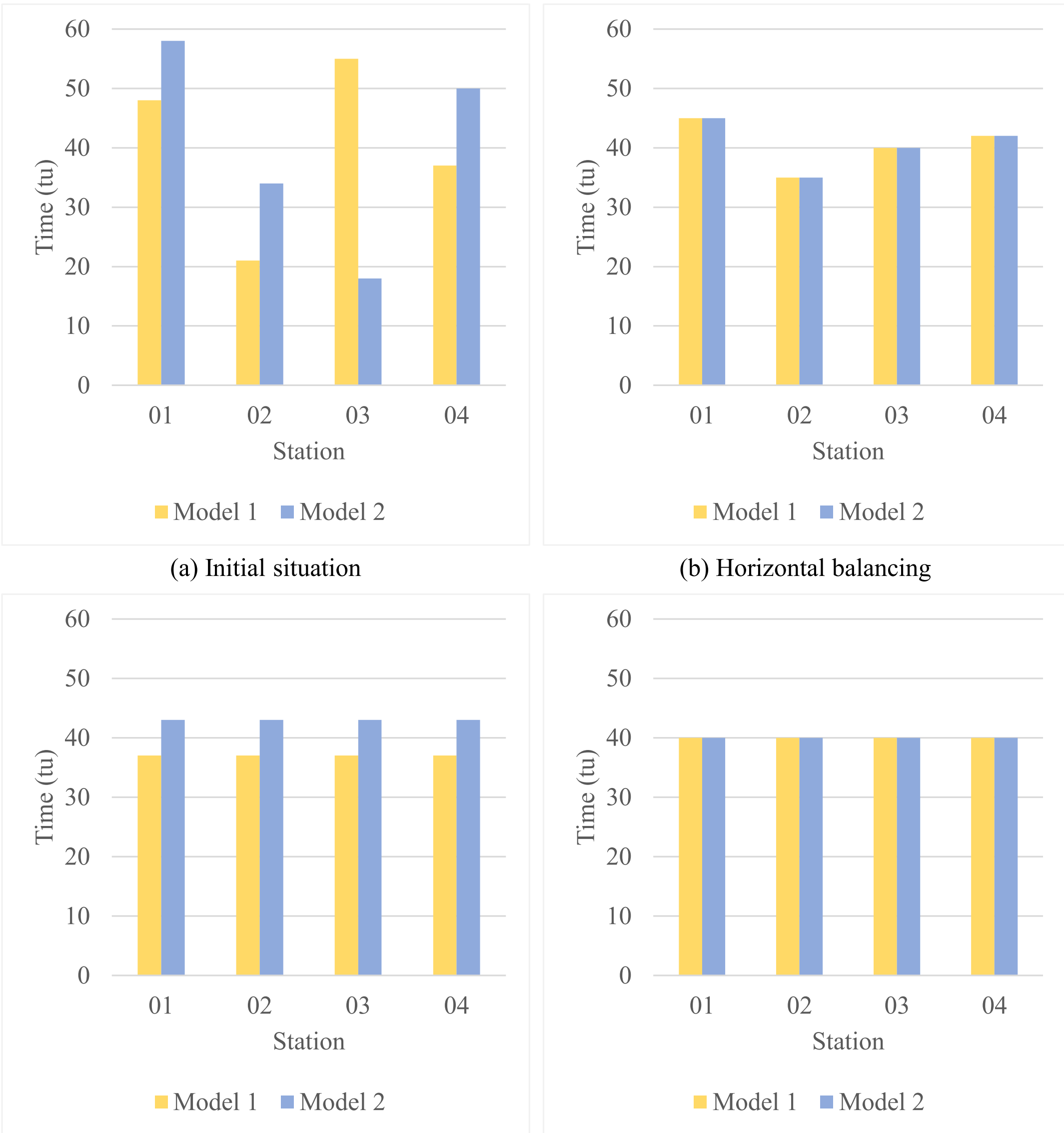
- Minimization of the time difference between stations or operators;
- Minimization of the smoothness index[2]
- Minimization of the cost of implementation;
- Maximization of the system efficiency.

[2] Indicates how close the balancing of different stations is, formally defined by Equation 5.

Although, in practice, the intention is to achieve several objectives, they are often conflicting. Thus, when improving a result, a possible loss in another occurs, then it is necessary to find an equilibrium condition for the studied situation.

Different products can be processed at the same workstation in the context of a mixed-model line. Because the processing times of each product at each station can differ, they consequently influence the workstation's workload. Provided that issue, the concepts of "vertical" and "horizontal" balancing arise, which are linked to a series of indicators for line balancing. The vertical balancing aims to balance the average times between the different stations of the line, considering the times of the different models in each station. The horizontal balancing, in a complementary way, seeks to balance the workload of a station for the different models produced (MERENGO; NAVA; POZZETTI, 1999). To clarify these concepts, Figure 5 shows an example.

Figure 5 – Example of horizontal balancing, vertical balancing, and both balanced simultaneously



(a) Initial situation (b) Horizontal balancing

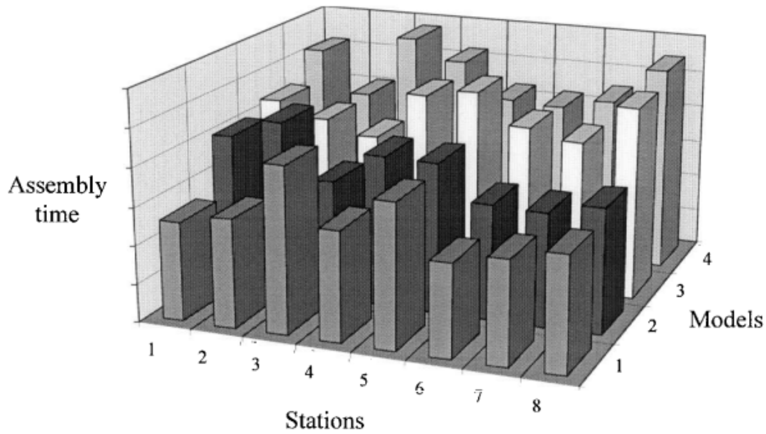
(c) Vertical balancing (d) Horizontal and Vertical balancing

Source: Own authorship (2023).

Balancing also helps to solve existing variations in production lines, which, according to (BUKCHIN, 1998), are two. Figure 6 illustrates the hereafter described variations:

- Model variability is generated due to the difficulty of obtaining a perfect balance for each model separately. It is defined as the variability of the assembly times of a given model assigned to different stations. This condition can be improved by vertical balancing;
- Station variability results from the line's different processing times, precedence, and technological constraints; it is defined as the variability of the assembly times of different models assigned to a specific workstation. Though in turn, it can be improved by horizontal balancing.

Figure 6 – Model and station variability



Source: (BUKCHIN, 1998).

In this context, there are different indicators to define operational bounds or to verify whether the objectives are being achieved, as described hereafter.

The lower bound for the minimum number of workstations (KRAJEWSKI, 2009) is defined using Equation (1). It is calculated by the sum of all tasks, divided by the takt time of the line. Then, this result is rounded up to the following integer number. In this equation, T indicates the total number of tasks to be performed, D_t the duration of each task t, and TKT the takt time of the line (time limit for the average productivity of the line to meet demand).

$$S_{min} = \left\lceil \frac{\sum_{t=1}^{T} D_t}{TKT} \right\rceil \tag{1}$$

Line Efficiency (LE) is the percentage of time the line is actually producing. It represents the ratio between the sum of activity times and the time the product remains on the line (BECKER; SCHOLL, 2006), according to Equation (2). In this equation, nS is the total number of stations, and CT is the cycle time.

$$LE\ (\%) = \frac{\sum_{t=1}^{T} D_t}{nS \cdot CT} \cdot 100 \tag{2}$$

Equation (3) represents the Idle Time (IT), which is the sum of the idle times of each station (EREL; SARIN, 1998). It is the total time that the product stays on the line minus the sum of the duration of tasks.

$$IT = nS \cdot CT - \sum_{t=1}^{T} D_t \tag{3}$$

Balancing Delay (BD), represented by Equation (4), is the line inefficiency due to workload imbalance (YIN; JIANG, 2016). Still, according to Yin & Jiang (2016), in practical terms, a balancing delay indicator of up to 10% is considered excellent, between 10% and 20% is considered good, and above 20% is considered bad.

$$BD\ (\%) = \frac{nS \cdot CT - \sum_{t=1}^{T} D_t}{nS \cdot CT}.100 \tag{4}$$

The Smoothness Index (SI), according to Becker & Scholl (2006), aims to measure the difference between the cycle time and the average time of each station as per Equation (5). The smaller the SI value (closer to zero), the more homogeneous the average workload tends to be between the different stations. ST_s represents the time of the station s.

$$SI = \sqrt{\sum_{s=1}^{S} (CT - ST_s)^2} \tag{5}$$

For assembly lines with more than one manufactured model, the Horizontal Smoothness Index (HSI) is used according to Equation (6), in order to evaluate the difference between cycle time and station times for each model (MERENGO; NAVA; POZZETTI, 1999). Similar to SI, the smaller the HSI values, the more homogeneous the workload between different models. In this equation, $ST_{s,k}$ and $ST_{s,m}$ are the station times for models k and m (M indicates the total number of models), and mx_k and mx_m the production mix for models k and m.

$$HSI = \sum_{s=1}^{S} \frac{\sqrt{\sum_{k=1}^{M} (ST_{s,k} - \sum_{m=1}^{M} ST_{s,m} \cdot mx_m)^2}}{\sum_{k=1}^{M} ST_{s,k} \cdot mx_k} \tag{6}$$

2.4 Sequencing of mixed-model assembly lines

Lines working with different products have a more complex problem, that in addition to assigning tasks to stations (balancing), need to define in which sequence the parts are going to be produced (sequencing) (SIKORA, 2022).

The sequencing of an assembly line aims to find the best product sequence to meet the model mix demand, while minimizing the total workload, idle times, station lengths, throughput times, or costs. It is a short-term decision problem, usually performed after the line balancing has already been executed (SCHOLL; KLEIN; DOMSCHKE, 1998). Unlike the balancing that is usually performed when designing the production line, the sequencing changes constantly according to demand fluctuations (SIKORA, 2021).

For sequencing purposes, it is usual to use the Minimum Part Set (MPS), which is the smallest set of parts that can be repeated until it reaches the desired productive demand. If the goal is to produce 60 parts of model A and 40 parts of model B, it is safe to say that the MPS could be of three parts of model A and two parts of model B, and it would be repeated 20 times until it meets the demand (LOPES, 2021).

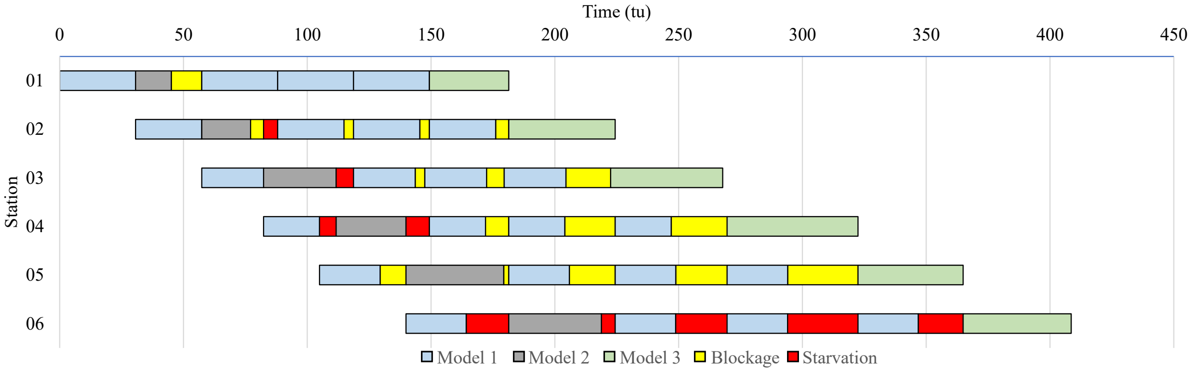
For mixed-model lines, where one single line has two or more different product variants, the number of tasks and the workload of each model may vary. Therefore, if variants with high working times are sequentially constructed, overload may result, and to fix this issue, line sequencing/scheduling can be used (MÄRZ, 2012).

Sequencing or scheduling assembly lines introduces a concept that did not exist in the balancing: the availability of stations. In an unpaced line, a given station may starve when waiting for the product from the previous station or be blocked when the station has finished its tasks but needs to wait for the next station to be available (CASTELLUCCI; COSTA, 2015).

In order to get a result of sequencing that is not worse than the balancing of a mixed-model assembly line, the work content of all models should be the same, and they should be evenly distributed among the stations in a way that there would be no blockage or starvation, so the sequence would not affect the performance (BUKCHIN, 1998). However, this is not a realistic condition. When performing the balancing of an assembly line, the station time is considered to be the processing time of its tasks allocated on the station, and the cycle time is the highest station time among the line. With the concepts of starvation and blockage, in addition to the processing time, the part can now be held in the station waiting to go to the next one, or a station can be free waiting for the part from the previous station. To consider that, a new variable called Steady-State Cycle Time (CTST) is used, which considers the difference between the time the last part left the station and the first part arrived at the station in a way that all idle times are considered, and the result is more accurate (LOPES, 2021).

Figure 7 shows a Gantt chart illustrating the sequencing of six parts of three different products, with the blockage in yellow and the starvation in red. After the second part is processed on station 01, it can not move to the second station since the latter is still processing part 1, which is considered a blockage. However, on station 04, after processing part 1, the station is idle because it is waiting for part 2 to be processed on station 03, called starvation.

Figure 7 – Example of Gantt chart



Source: Own authorship (2023).

Most of the authors perform the balancing and the sequencing independently because of computational constraints (MEIRA, 2015). However, performing them simultaneously leads to better results than independently or sequentially (BOYSEN; FLIEDNER; SCHOLL, 2009b).

Some authors perform both the balancing and sequencing at the same time, but they consider the cycle time as a parameter: (KUCUKKOC; ZHANG, 2016), (NILAKANTAN *et al.*, 2017), (ZHONG, 2017), (AKPINAR; ELMI; BEKTAŞ, 2017), (DELICE *et al.*, 2017), (DEFERSHA; MOHEBALIZADEHGASHTI, 2018), (DONG; ZHANG; XIAO, 2018). On the other hand, Lopes *et al.* (2020a) developed a model with three degrees of freedom to balance, sequence, and place buffers on an assembly line in an integrated manner. The authors also exploited the concept of steady-state cycle time. Thus, the work of (LOPES *et al.*, 2020a) is hereafter used as a modeling base in the present study.

Chapter 3 details the addressed problem in order to clarify that balancing and sequencing aspects are relevant to obtain optimized productive conditions.

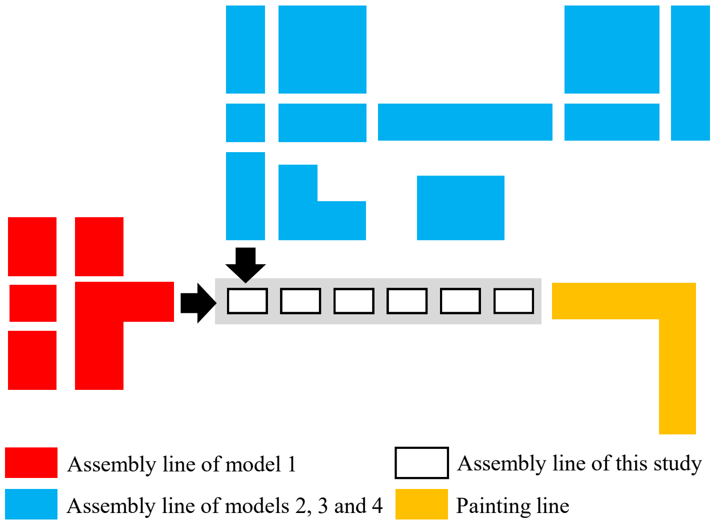
3 PROBLEM DESCRIPTION

This chapter presents the addressed problem, which involves the balancing and sequencing of an assembly line of automotive vehicles. First, the studied line is described according to its characteristics, and the products made on the line are introduced (Section 3.1). The precedence diagram and the current workload distribution are shown in Section 3.2. Finally, the production mix is presented (Section 3.3).

3.1 Assembly line and products

The assembly line of this study is considered a serial assembly line, where two different dedicated lines meet to form the first mixed-model stretch of the entire production line. Figure 8 shows a representation of the factory. It is important to notice that the assembly line of models 2, 3, and 4, represented in blue, just has one variant at this productive phase, so the three different models are only distinguished when they get into the line of this study.

Figure 8 – Simplified schematic representation of the factory

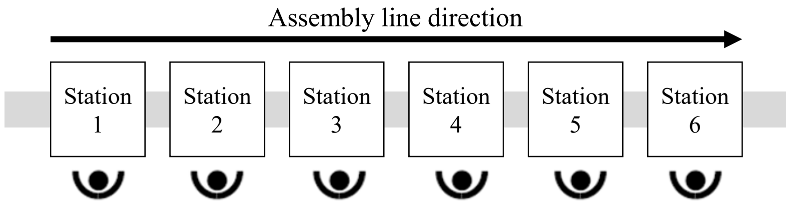


Source: Adapted from (MEIRA, 2015).

The studied line is unpaced and asynchronous, using a stop-and-go system on a conveyor belt, meaning that the product can only move to the next station once the operation in the current station is done, and if the next station is free to receive the part (product). The line has six operators working in pairs in three different stations, one on each side. However, to simplify the model, for balancing purposes the productive line is considered as six sequential stations, as tasks from the left and right side of the same station are independently done. In order to do that, the precedence diagram was designed in a way that respects the tasks dedicated on the left side and on the right side of stations, along with the set of customized restrictions. Further details are given on the mathematical model on Subsection 4.1.3.

An illustration of the serial layout considered with six stations is present in Figure 9. As a simplification hypothesis, deterministic task times were considered as the line operates with standardized mounting procedures and workers are previously trained to perform the tasks in a more homogeneous way. Indeed, in the automotive industry, it is usual to have standard task times for operations (BOYSEN; FLIEDNER; SCHOLL, 2008).

Figure 9 – Schematic representation of the considered assembly line



Source: Own authorship (2023).

The line works in two shifts, producing four different models with six operators fixed in stations. It is the first part of the factory where the mixed-model concept is employed, and this sector is responsible for the assembly of the doors, hood, fender, tailgate, and some spot welding points. At the moment of the study, the sector was the bottleneck of the entire assembly line. Each model's total workload (values in time units, tu) is present in Table 2. It is possible to notice that different workloads exist as a consequence of the quantity and complexity of the activities for the different models.

Table 2 – Workload for each model

Model	Total workload (tu)
Model 1	153.8
Model 2	259.2
Model 3	168.7
Model 4	249.5

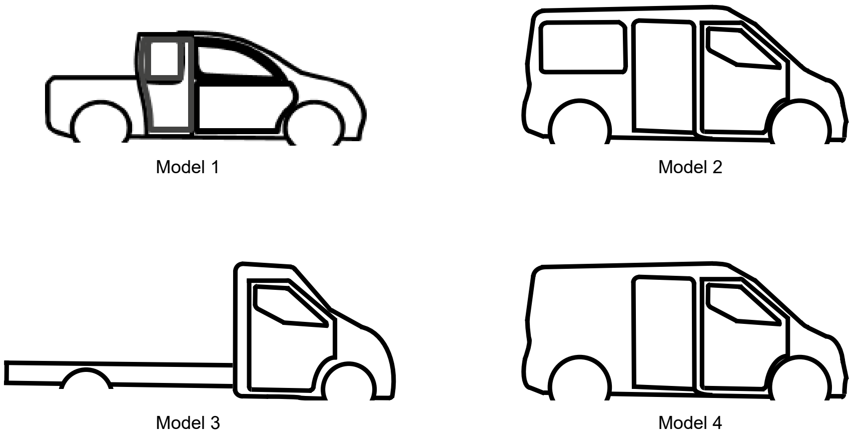
Source: Own authorship (2023).

Model 1 is a pickup, which requires some welding, part sanding before the assembly of the components, adjustment of gap and flush of the doors (frontal right, frontal left, back right, back left), hood, fender, tailgate, and fuel door assembly.

Models 2, 3, and 4 are vans. They all need part sanding before the assembly of the components. Similarly to model 1, they have the assembly and adjustment of the frontal doors. However, some differences are found in the case of the two back doors and lateral door: Model 3 is a chassis cab van and does not have those three doors mentioned, meaning that its time is shorter than the other two, whereas both Model 2 and 4 do have the three doors. In addition, the considered line also assembles some reinforcement parts, hood, and fender. A representation of models 1 through 4 can be seen in Figure 10.

In this assembly line, 190 tasks are performed with some particularities: some activities are fixed to stations, accordingly to physical restrictions of the factory installation; some tasks that are incompatible with each other (like the assembly of the right door cannot be performed at the same station of the assembly of the left door), that is, that cannot be performed at the

Figure 10 – Schematic representation of the different models



Source: Own authorship (2023).

same station; tasks that should respect a given distance between stations (like the screwing of the door that must be done at the same station as the assembly of the door, because if it moves to the next station without being fixed, it might fall); and tasks that should be performed before others, in other words, that precede other activities (such as regulate the door must be after the door assembly).

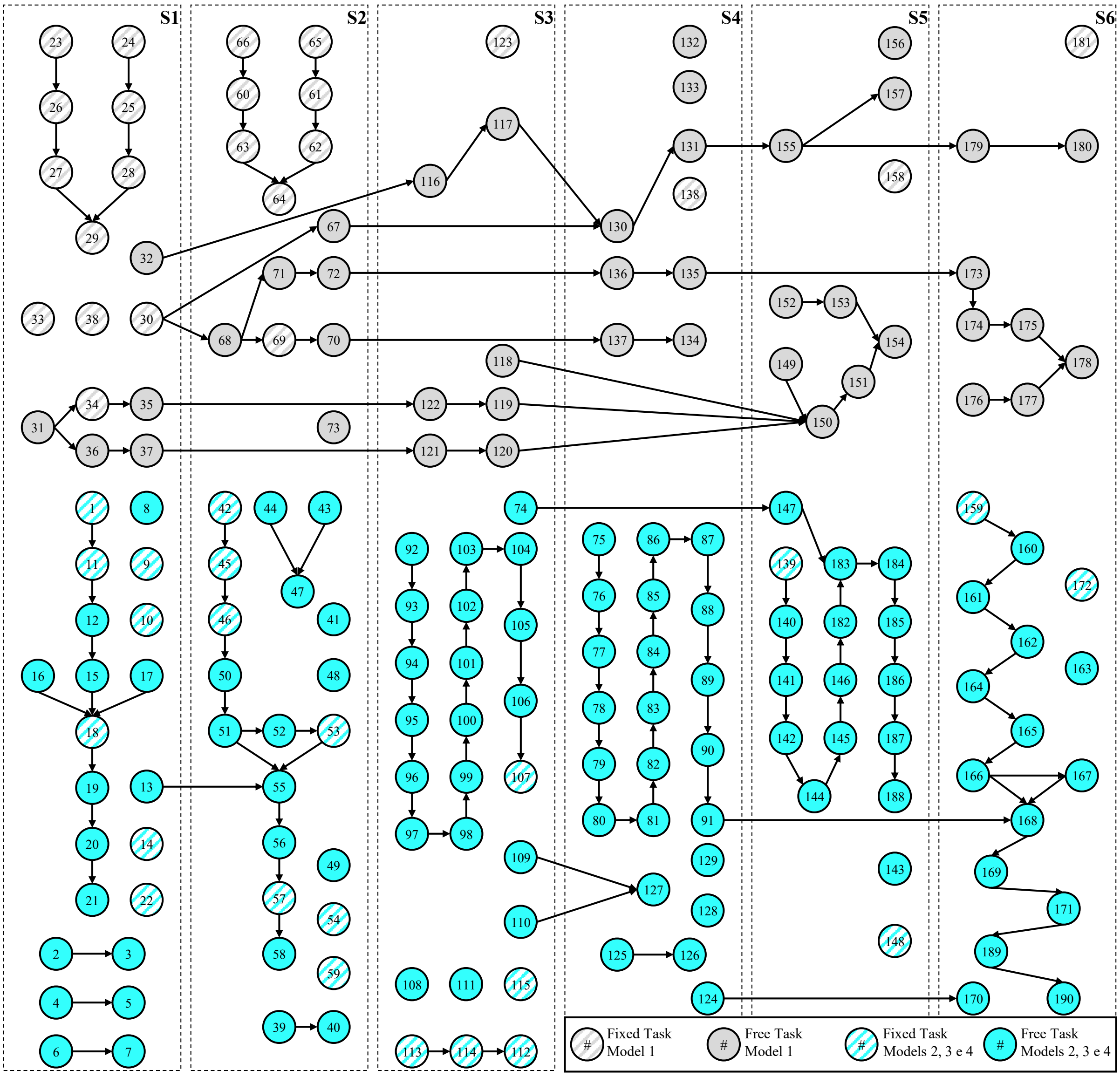
3.2 Precedence diagram and current workload

The initial allocation of tasks at stations (S1 to S6), shown in Figure 11, presents the precedence relationships between tasks and fixed tasks, being separated for Model 1 and Models 2, 3, and 4 because of the similarity of the three models and their tasks in common. The detailed list of each task and the respective time can be found in Appendix A.

In a theoretical view, a type-F balancing problem is characterized by a feasibility problem, given a fixed number of stations and a known takt time (TKT). This is strictly related to the considered problem, as the number of stations is given and a known TKT is imposed, but the known TKT undergoes changes with a certain frequency. Therefore, the interest is knowing the smallest possible cycle time that still respects a given TKT. Then, the balancing problem is characterized as type-2 (cycle time minimization) in order to determine the smallest cycle time given the available resources of the line.

The current workload distribution for the six stations (or operators as each one is fixed to a station) can be seen in Figure 12. The cycle time exceeds the currently considered takt time in three stations, which means that the line is not able to supply the demand. At stations 01, 02, and 04, the workload is 7.6%, 8.8%, and 3.7% above takt time, while at stations 05 and 06, there is an idleness of 6.08 tu and 6.72 tu, which represents 19.1% and 21.1% of the takt time, respectively. Thus, the aim is to obtain a better use of resources in order to try to supply the

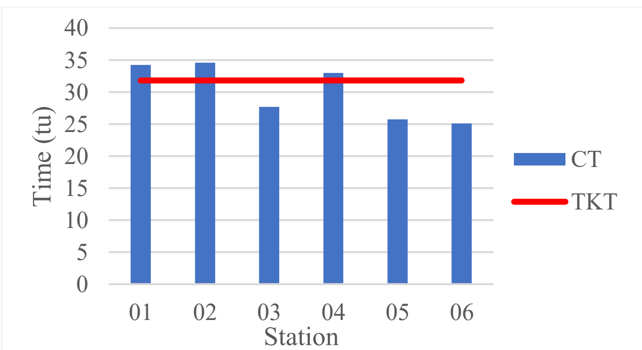
Figure 11 – Assembly line precedence diagram showing fixed tasks and initial station placement



Source: Own authorship (2023).

demand without making investments with the opening of new workstations, since according to Equation (1), the minimum number of stations required for production is equal to six.

Figure 12 – Current cycle time of line stations



Source: Own authorship (2023).

3.3 Mix of production

This study considered six months of production, distributed in eight different mixes. The mix usually varies over time since the factory only produces after the customer's order. Different takt times can be observed along the considered months. Additionally, some external factors also have an influence, such as the availability of components, the launch of a new product, and the end of life of an old product.

The takt time is affected by the number of products to be delivered to the customer and the available production time, which means that the quantity of shifts directly influences this parameter.

Table 3 shows the product distribution according to the eight specific mixes practiced during the 6-month period analyzed. Each mix was produced for about 3 to 4 weeks, and it could change with the customer demand, shortage of raw materials of one model, ramp-up of new products, or end of production of a model. The takt time is a parameter defined by the company according to the increase or drop in expected production volumes. The table also brings in the last line the equivalent mix adopted for the considered 6-month period, which is used for the first analysis of this study (Analysis 1 in Chapter 4 and Chapter 5).

Table 2 evidences the workload differences among the four models; this information, in conjunction with the variability of mixes in Table 3, indicates that the necessary takt time has to vary according to the considered productive mix.

Table 3 – Mix and TKT (tu) considered

Mix	Model 1 (%)	Model 2 (%)	Model 3 (%)	Model 4 (%)	TKT (tu)
Mix 1	70	20	5	5	31.83
Mix 2	68	24	2	6	31.83
Mix 3	65	15	10	10	31.83
Mix 4	65	20	10	5	31.83
Mix 5	45	40	5	10	51.16
Mix 6	50	20	10	20	59.46
Mix 7	60	16	8	16	44.61
Mix 8	70	15	10	5	33.08
Equivalent 6 months	60	20	10	10	40.37

Source: Own authorship (2023).

Based on the practical aspects given in the presented chapter, Chapter 4 brings the methodology proposed to address the practical problem. These aspects indicate that, for instance, significant temporal differences among the four productive models exist (Table 2), an equivalent productive mix can be characterized for the 6-month period (Table 3), but significant productive changes may be necessary according to the mix, raising the question of whether an equivalent mix can be used to obtain the productive balancing, a fact clarified in Chapter 5, results.

4 METHODOLOGY BASED ON MATHEMATICAL MODELS

This chapter presents the methodology proposed to solve the practical production line problem (Section 4.1). Balancing aspects are initially considered, but sequencing aspects are also exploited owing to the significantly different production mixes (*e.g.*, Table 3). This methodology is centered on MILP models. The detailed formulations for the proposed mathematical models are presented in Section 4.2.

4.1 Methodology

In this section, the proposed methodology is presented, which is centered on the resolution of MILP models. Line balancing performed by MILP can return the optimal solution for a specific situation represented in the model through a data set. Nevertheless, numerous changes in the allocation of tasks may be necessary for such a solution, usually accompanied by training for line and logistic operators, layout changes that require significant investments, and the requirement of tools and/or machines. In practice, such activities may not be simple and require a certain amount of effort from the team involved. Therefore, it would not be feasible to carry out a new line balancing with each update of the factory production planning.

Because the factory in question presents a significant variation, both in the number of vehicles produced and in the partial demand of the different models, it becomes necessary to find a balancing proposal that allows the fulfillment of the indicators in different scenarios, that is, a “resilient” proposal in the face of varying production conditions.

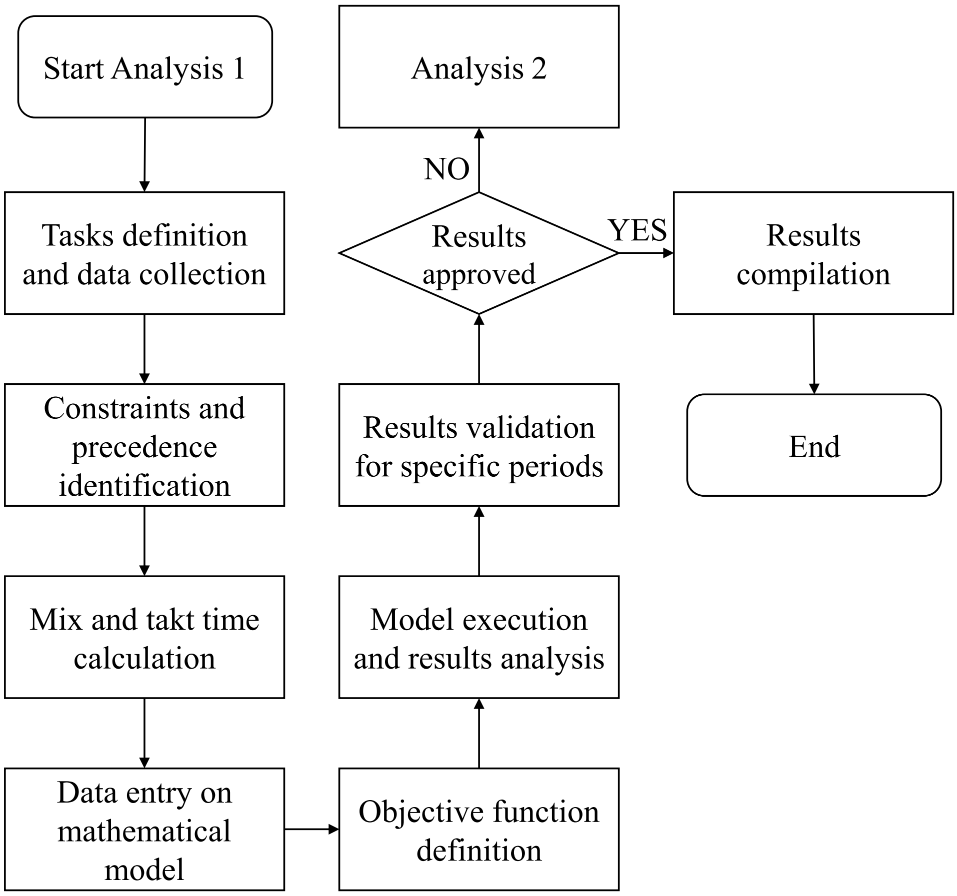
The study is conducted in three analyses. The first brings an equivalent demand for six months (Analysis 1). The second treats separate periods of time to search for a solution capable of meeting the different scenarios (Analysis 2). The last one expands the model to consider a balancing and sequencing problem (Analysis 3), which yielded a more complex modeling formulation but with the potential of finding a more accurate result concerning the synergy of balancing and sequencing. These three analyses were developed as the problem’s characteristics were deepened.

4.1.1 Analysis 1

The first analysis considers six months of production, using an equivalent mix and takt time. Analysis 1 looks for balancing tasks between the six stations to improve the efficiency and output of products. The methodology employed can be seen in Figure 13.

The first step is the definition of tasks performed on the assembly line and the execution of *in loco* data collection. There is a total of 598 activities, but to improve the model’s processing time, and also to complain with quality requirements that indicate that some activities have to

Figure 13 – Methodology used in Analysis 1



Source: Own authorship (2023).

be executed in sequence, those tasks are grouped, resulting in a total of 190 tasks (or blocks of tasks).

In the following step, the restrictions and particular precedence of the line are identified and implemented on the model in a way that represents the assembly line. There are restrictions regarding incompatible tasks, distances between tasks, fixed tasks, and tasks that should be performed after others.

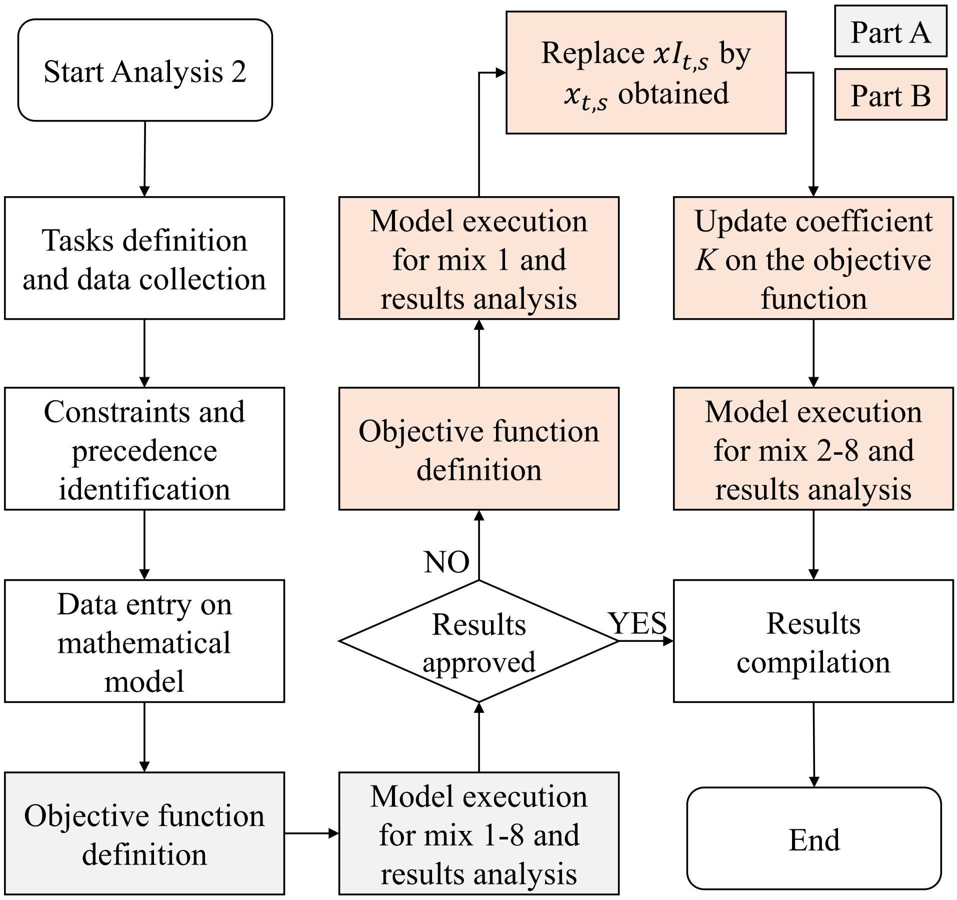
Next, the equivalent mix and the takt time are calculated for the 27 weeks of production. For the mix, the number of products for each of the four models was added and divided by the total vehicle demand. For the take time, the production's available time for the entire period was divided by the total product demand. In the sequence, these data are provided to the mathematical model. After this, a step is performed to define the objective function. For this, some preliminary tests are executed using different objective functions to compare the results. It also allowed to find a reference of the best result for the mix regarding the CT, the horizontal balancing, and the vertical balancing.

The model is then executed, and the next step is to validate the results for each specific period, confirming whether the proposal would meet the periods analyzed individually. For this, the balance obtained was used with the distributions of eight different mixes.

4.1.2 Analysis 2

A second analysis is developed, but now, looking at each period separately. During the six months of analyses, the production mix changed eight times, defining eight different production mixes, named mix 1 to mix 8. Therefore, eight periods of time, which faced different production mixes, are hereafter referred to. The methodology is presented in Figure 14. This analysis is divided into two parts: Part A, which does not consider the number of task changes between the solutions, and Part B, which aims to minimize the number of task changes.

Figure 14 – Methodology used in Analysis 2



Source: Own authorship (2023).

The data collection, constraints, and precedence are reused from the *in loco* observations already made for Analysis 1. However, the implementation in the mathematical model took into account each period individually, and the number of changes compared to the previous result is evaluated.

Initially, an analysis is performed without considering the number of changes in tasks compared to the initial allocation or from one period to the next (Part A). Later, to avoid changing the task allocation for each mix of production, a variable concerning the weight of the changes is introduced to the objective function, aiming to decrease the number of changes. A study is performed to choose this variable's coefficient on the objective function.

Once a balancing proposal is found, the allocation of tasks from the initial solution $(xI_{t,s})$ is replaced by the allocation of tasks obtained ($x_{t,s}$), which is called Proposal Analysis 2. Then, the coefficient K of the objective function is changed to verify if the takt time would be respected for different product mixes and the necessary amount of activity changes compared to the previous allocation.

In this context, different production conditions (productive mix) may exist, each with a respective takt time required. At this point in the analysis, task changes are considered with greater emphasis when a productive mix different from Part A of Analysis 2 (reference value) is evaluated, but not preventing changes from being carried out, if necessary, to reach the takt time associated with the mix.

Subsequently, the production mix data is updated, and the model is executed as many times as necessary until the analyses of the different mixes involved are completed, finally arriving at the compilation of the results.

4.1.3 Analysis 3

The next step is to add the sequencing feature to the model. To do that, the Equations and Inequalities (25)-(37) are added to the mathematical formulation (Section 4.2). Similar to Subsection 4.1.2, Subsection 4.1.3 has two parts, the first without considering the number of changes of tasks and the second one considering the changes. Figure 15 shows the methodology employed in this analysis.

The first step is to find the MPS for the different mixes, which reduces the problem's complexity in computational terms and returns smaller sequences of products that are easier to replicate. Then the model is run, fixing the initial allocation of tasks in a way that there would be no balancing but purely sequencing of the models to get the best result for the $CTST$ (Steady-State Cycle Time), so we could get a reference value for this new indicator, using Equation (38).

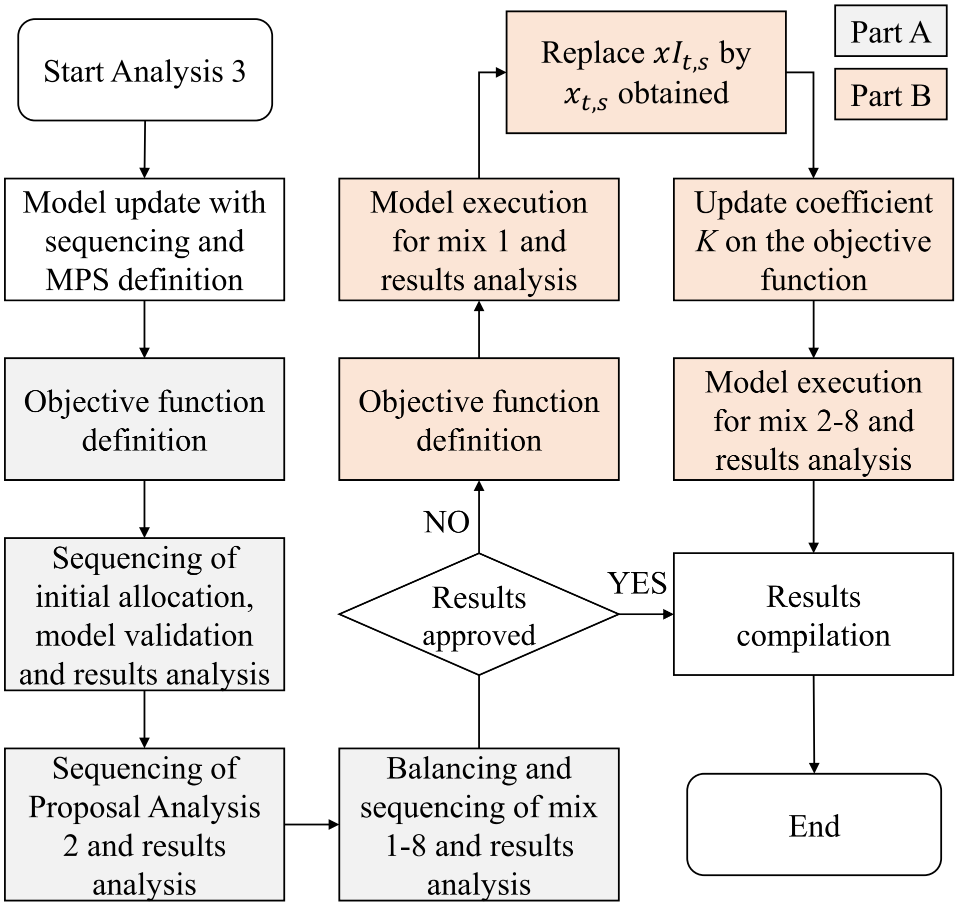
A study is performed to evaluate the difference between CT and $CTST$, only for one model, to confirm whether the result would be the same as it was supposed to be and validate the model. After this, the model is run for the balancing obtained on Proposal Analysis 2, fixing the proposed allocation of tasks, again to perform only the optimal sequencing given the balancing made previously.

Afterwards, the model is executed again, but now with the freedom to balance and sequence. Later, the same reasoning of Subsection 4.1.2 Part B is performed, considering the number of task changes. The objective function is updated to Equation (39) for this.

While analyzing the results, some areas of opportunity are identified. First, to perform horizontal and vertical balancing, using the variables[1] MDS (Maximum difference between station time and average time) and $MDSM_m$ (Maximum difference between station time and average time of each model m). Second, by balancing the idle time between the operators, the new varia-

[1] The entire set of used variables are afterwards defined in Table 7.

Figure 15 – Methodology used in Analysis 3



Source: Own authorship (2023).

ble $MaxIT$ (Maximum idle time among all parts and stations) is tested to limit significant waiting times between two consecutive parts. Equation (40) is used to verify this condition.

4.2 Mathematical model

The mathematical model used in this study is based on the model proposed by Lopes *et al.* (2020a). Extensions proposed are herewith detailed in the present section. To implement and execute the model, variables, parameters, sets, and indexes are defined. The indexes can be found in Table 4. The parameters are defined in Table 5. Then, the sets are displayed in Table 6. Finally, the variables used in the model are presented in Table 7.

Table 4 – Indexes used in the model

Index	Description	
t	Task index t	1...T
s	Station index s	1...S
m	Model index m	1...M
p	Part sequence index p	1...P

Source: Own authorship (2023).

Table 5 – Parameters used in the model

Parameter	Description
Ch_t	Weight change for each task t
D_t	Duration of task t pondered by the production mix
$DM_{t,m}$	Duration of task t for each model m (in tu)
nS	Total number of stations
mx_m	Demand of each model m
TKT	Takt time (in tu) (see Subsection 2.1.1)
$Prod_m$	Number of produced parts for each model m
nP	Total number of produced parts
$xI_{t,s}$	Initial allocation of tasks
C_{CT}	Coefficient of the objective function associated to CT (see Table 7)
C_{MDS}	Coefficient of the objective function associated to MDS (see Table 7)
C_{MDSM_m}	Coefficient of the objective function associated to $MDSM_m$ (see Table 7)
C_{WCh}	Coefficient of the objective function associated to WCh (see Table 7)
K	Coefficient to change the weight of the variable WCh to reduce the number of changes
C_{CTST}	Coefficient of the objective function associated to $CTST$ (see Table 7)
C_{MDSseq}	Coefficient of the objective function associated to MDS for the sequencing part
$C_{MDSMseq}$	Coefficient of the objective function associated to $MDSM_m$ for the sequencing part
C_{MaxIT}	Coefficient of the objective function associated to $MaxIT$ (see Table 7)
$BigM$	A sufficient large number

Source: Own authorship (2023).

Table 6 – Sets used in the model

Set	Description
T	Set of tasks t
S	Set of stations s
M	Set of models m
PC	Set of task precedence relations
FT	Set of tasks fixed to stations
I	Set of tasks that cannot be placed in the same station (incompatible)
D	Set of tasks separated by a distance of d stations
xI	Set of tasks t of station s on the initial allocation
P	Set of produced parts p during sequencing

Source: Own authorship (2023).

4.2.1 Balancing model

To balance the line, seeking to increase the productive capacity, a model in MILP is developed according to Equations (7)-(24). To determine the objective function, some preliminary tests were performed with different indicators. Firstly, it was evaluated by minimizing the CT (alternative a), then minimizing the MDS (alternative b) to have a horizontal balancing, later minimizing the $MDSM_m$ (alternative c) to have a vertical balancing, and the last alternative considered all three mentioned variables (alternative d) to try to look for both, horizontal and vertical balancing together. Equation (7) is the objective function chosen[2] for Analysis 1 and the first part of Analysis 2. The parameters C_{CT}, C_{MDS}, and C_{MDSM_m} are weighting factors. The order of magnitude defined for these factors is $C_{CT} \gg C_{MDS} \approx C_{MDSM_m}$.

[2] Afterwards, Section 5.1 brings a numerical analysis that justifies this choice for the objective function.

Table 7 – Variables used in the model

Variable	Description
CT	Line cycle time (see Subsection 2.1.1)
$CTST$	Cycle time of steady-state (see Section 2.4)
$x_{t,s}$	Binary variable of allocation of task t on station s.
	Has value 1 if the task t is allocated on station s, otherwise is set to 0.
ST_s	Station time (working time) of station s
MDS	Maximum difference between station time and average time
avS	Average time of stations
$STM_{s,m}$	Time of station s for each model m
$avSM_m$	Average time of stations for each model m
$MDSM_m$	Maximum difference between station time and average time of each model m
WCh	Weight of all changes regarding the previous allocation of tasks
$Seq_{m,p}$	Binary variable of sequencing of model m being produced on the sequence of parts p.
	Has value 1 if the model m is produced on sequence number p, otherwise is set to 0.
$Tin_{p,s}$	Time that part p enters the station s
$Tx_{p,s}$	Processing time of part p on station s
$Tout_{p,s}$	Time that part p leaves the station s
$StarvationP_{p,s}$	Starvation of part p on station s
$BlockageP_{p,s}$	Blockage of part p on station s
$IdleTimeP_{p,s}$	Total idle time of part p on station s
$MaxIT$	Maximum idle time among all parts and stations

Source: Own authorship (2023).

$$\text{Minimize } z = C_{CT} \cdot CT + C_{MDS} \cdot MDS + C_{MDSM_m} \cdot \sum_{m=1}^{M} MDSM_m \tag{7}$$

The constraints used in the balancing model are presented from Equation (8) to Equation (23). Equation (8) indicates that each task must be allocated in a station, respecting the precedence relations between tasks, Inequality (9). There are fixed tasks at stations, according to the physical constraints of the line, Equation (10). Additionally, there may be incompatibility for allocating tasks in the same station (11) and distance (in stations) between tasks, Equation (12).

$$\sum_{s=1}^{S} x_{t,s} = 1, \quad \forall t \in T \tag{8}$$

$$\sum_{s=1}^{S} s \cdot x_{t1,s} \leq \sum_{s=1}^{S} s \cdot x_{t2,s}, \quad \forall (t1,t2) \in PC \tag{9}$$

$$x_{t,s} = 1, \quad \forall (t,s) \in FT \tag{10}$$

$$x_{t1,s} + x_{t2,s} \leq 1, \quad \forall s \in S, \forall (t1,t2) \in I \tag{11}$$

$$\sum_{s=1}^{S} s \cdot x_{t1,s} + d = \sum_{s=1}^{S} s \cdot x_{t2,s}, \quad \forall (t1,t2,d) \in D \tag{12}$$

Inequality (11) was important to aid in simplifying the problem from a two-sided assembly line to a simple assembly line. The set I was carefully defined in a way that the inequality stated that tasks that must be done on the left side are incompatible with tasks that must be done on the right side. Also, Equation (12) and the set D were used to allocate tasks with a "distance of one station", such as the assembly of the left and right door, in a way that if the right door is assembled on station 1, the left door will be assembled on station 2. This rationale was combined with the customized set of precedence between tasks to respect the parallelism of the line and simulate its two-sided aspect.

The time of each station is equal to the sum of tasks allocated to it, Equation (13). This time has to respect the takt time, as indicated by Inequality (14), if demand requirements can be supplied; otherwise, this inequality has to be omitted for not causing an infeasible condition. For each mix, a constant average station time is defined by the total station time divided by the number of stations, Equation (15). The maximum difference between the time of the stations and the average time is represented by Inequalities (16) and (17).

$$ST_s = \sum_{t=1}^{T} x_{t,s} \cdot D_t, \quad \forall s \in S \tag{13}$$

$$ST_s \leq TKT, \quad \forall s \in S \tag{14}$$

$$avS = \frac{\sum_{s=1}^{S} ST_s}{nS} \tag{15}$$

$$MDS \geq ST_s - avS, \quad \forall s \in S \tag{16}$$

$$MDS \geq -ST_s + avS, \quad \forall s \in S \tag{17}$$

For horizontal balancing, Equations (18) and (19) and Inequalities (20) and (21) are similar to Equations (13) and (15) and Inequalities (16) and (17), but for each model.

$$STM_{s,m} = \sum_{t=1}^{T} x_{t,s} \cdot DM_{t,m}, \quad \forall s \in S, \forall m \in M \tag{18}$$

$$avSM_m = \frac{\sum_{s=1}^{S} STM_{s,m}}{nS}, \quad \forall m \in M \tag{19}$$

$$MDSM_m \geq STM_{s,m} - avSM_m, \quad \forall s \in S, \forall m \in M \tag{20}$$

$$MDSM_m \geq -STM_{s,m} + avSM_m, \quad \forall s \in S, \forall m \in M \tag{21}$$

Inequality (22) indicates that the cycle time is at least the largest time among stations.

$$\sum_{t=1}^{T} x_{t,s} \cdot D_t \leq CT, \quad \forall s \in S \tag{22}$$

In the second part of Analysis 2, the number of changes is evaluated. The relative weight of changes is obtained by Equation (23). As an example, the Ch_t parameter used for the studied case is 1 for simple changes (with no changes in logistical supply or tools and short duration of operator training), 3 for changes with medium complexity (with changes in logistical supply and moderate duration of training), and 5 for highly complex changes (with tooling changes, high investment costs and training considered long).

$$WCh = \sum_{t=1}^{T} \sum_{s=1 \mid xI_{t,s}=1}^{S} Ch_t \cdot (1 - x_{t,s}) \tag{23}$$

The objective function considering now the WCh is found in Equation (24). The parameter C_{WCh} is a weighting factor for this term in the objective function. Also, the parameter K is a coefficient that will change the weight of the variable WCh, aiming to reduce the number of task changes, depending on the methodology part, as specified in Section 4.1. The order of magnitude defined for these factors is $C_{CT} \gg C_{MDS} \approx C_{MDSM_m} \approx C_{WCh}$.

$$\text{Minimize } z = C_{CT} \cdot CT + C_{MDS} \cdot MDS + C_{MDSM_m} \cdot \sum_{m=1}^{M} MDSM_m + K \cdot C_{WCh} \cdot WCh \tag{24}$$

4.2.2 Balancing and sequencing model

When the sequencing feature is considered in the model, Equation (25) to Equation (40) are added. Equations (25) to (32) are from Lopes *et al.* (2020b), while Equations (33)-(40) are proposed for this study. Inequality (14) is not being considered for the integrated balancing-sequencing analysis.

Equation (25) states that the number of sequenced parts of each model m equals the number of produced parts of each model m. Equation (26) indicates that only one model can be sequenced at a slot dedicated to part p.

$$\sum_{p=1}^{P} Seq_{m,p} = Prod_m, \quad \forall m \in M \tag{25}$$

$$\sum_{m=1}^{M} Seq_{m,p} = 1, \quad \forall p \in P \tag{26}$$

Inequality (27) indicates that part p can only leave the station s after it enters and is processed. Inequalities (28) and (29) determine the processing time of the part. If the binary variable $Seq_{m,p}$ is equal to 1, then the $Tx_{p,s}$ will be equal to $STM_{s,m}$.

$$Tout_{p,s} \geq Tin_{p,s} + Tx_{p,s}, \quad \forall s \in S, \forall p \in P \tag{27}$$

$$Tx_{p,s} \geq STM_{s,m} - BigM \cdot (1 - Seq_{m,p}), \quad \forall s \in S, \forall p \in P, \forall m \in M \tag{28}$$

$$Tx_{p,s} \leq STM_{s,m} + BigM \cdot (1 - Seq_{m,p}), \quad \forall s \in S, \forall p \in P, \forall m \in M \tag{29}$$

It is considered that part p enters the station s immediately when it leaves station $s-1$, as indicated by Equation (30).

$$Tin_{p,s} = Tout_{p,s-1}, \quad \forall p \in P, \forall s \in S \,|\, s > 1 \tag{30}$$

Part p can only leave station s after the previous part $p-1$ leaves the next station $s+1$, as per Equation (31).

$$Tout_{p,s} \geq Tout_{p-1,s+1}, \quad \forall p \in P \,|\, p > 1, \forall s \in S \,|\, s < nS \tag{31}$$

Part p enters station s after the previous part $p-1$ leaves station s, as per Equation (32).

$$Tin_{p,s} \geq Tout_{p-1,s}, \quad \forall p \in P \,|\, p > 1, \forall s \in S \tag{32}$$

In addition to the sequencing, the idle times originated from starvation and blockage are calculated and defined as per Equations (33)-(34). The starvation is the station's downtime because it is waiting for parts from the previous station. It is calculated by the difference between the time part $p+1$ enters the station and the time part p leaves it. For instance, if part p leaves station s on time 100 tu, and the next part $p+1$ enters station s on time 130 tu, it means that the station s is starved for 30 tu. By definition it was considered that the starvation when $p = nP$ is zero.

$$StarvationP_{p,s} = Tin_{p+1,s} - Tout_{p,s}, \quad \forall p \in P \,|\, p < nP, \forall s \in S \tag{33}$$

Blockage is the wait at the station s because the next station, $s+1$, is busy. It is calculated by the difference between the time that the part left the station and the sum of the time it entered the station and its processing time. For example, if a part enters on the time 100 tu, it is processed during 30 tu, but only left the station on time 150 tu, it means that it had a blockage of 20 tu on that station.

$$BlockageP_{p,s} = Tout_{p,s} - Tin_{p,s} - Tx_{p,s}, \quad \forall p \in P, \forall s \in S \tag{34}$$

The $IdleTimeP_{p,s}$ is considered as the sum of the starvation and blockage of each part, that is, the sum of $StarvationP_{p,s}$ and $BlockageP_{p,s}$, as defined per Equation (35).

$$IdleTimeP_{p,s} = StarvationP_{p,s} + BlockageP_{p,s}, \quad \forall p \in, \forall s \in S \tag{35}$$

The $MaxIT$ is the largest idle time between two parts in a station and is defined by Inequality (36). The smaller the $MaxIT$, the better the balancing between the stations' workload, so it is used to compare different solutions. Some other variables were preliminary considered to compare the solutions, such as the lead time, which would be the difference in time between the last part leaving the last station and the first part entering the first station, or the sum of all the idle times in all stations. Still, they were not considered appropriate since the mixes have different MPS. Therefore, the comparison between those indicators would not consider the same conditions. Alternatively, $MaxIT$ was defined and used.

$$MaxIT \geq IdleTimeP_{p,s}, \quad \forall p \in P, \forall s \in S \tag{36}$$

The steady-state cycle time represents the time at which parts will be produced while they are sequenced.

$$nP \cdot CTST + Tin_{1,s} \geq Tout_{nP,s}, \quad \forall s \in S \tag{37}$$

The objective functions used are described from Equation (38) to Equation (40). Equation (38) brings the first objective function used for the sequencing, aiming to minimize purely the steady-state cycle time.

$$\text{Minimize } z = CTST \tag{38}$$

Aiming to minimize the number of changes, as done in Subsection 4.1.2, Equation (39) includes the weighted changes WCh.

$$\text{Minimize } z = C_{CTST} \cdot CTST + K \cdot C_{WCh} \cdot WCh \tag{39}$$

Equation (40) is the final objective function that also considers in an integrated manner the MDS, $MDSM_m$, and $MaxIT$ variables. The order of magnitude defined for the weighting factors is $C_{CTST} \gg C_{WCh} > C_{MDSseq} \approx C_{MDSMseq} \approx C_{MaxIT}$.

$$\text{Minimize } z = C_{CTST} \cdot CTST + K \cdot C_{WCh} \cdot WCh + C_{MDSseq} \cdot MDS + C_{MDSMseq} \cdot \sum_{m=1}^{M} MDSM_m + C_{MaxIT} \cdot MaxIT \quad (40)$$

The methodology, which involves the presented set of equations and inequalities, is used in Chapter 5 for the different analyses.

5 RESULTS AND DISCUSSIONS

This chapter brings in Section 5.1 the results of the mathematical model for Analysis 1 with the balancing of a six-month period. Following, in Section 5.2 Analysis 2 obtains a balancing solution that minimizes the changes of tasks between the periods. Practical results are obtained from Section 5.2. Finally, Section 5.3 shares Analysis 3 with balancing and sequencing together, aiming to have a theoretical result that represents a balancing-sequencing problem in a serial arrangement. Analysis 3 highlights the potential differences between just balancing and balancing and sequencing a serial line.

The implementation and resolution are performed in the computational modeling and execution environment IBM ILOG CPLEX Optimization Studio 12.10, with a limited execution time of 60 minutes. A computer with a Core i5-8265U processor (1.80 GHz) and 8 GB of RAM is used. The mathematical model converged to the optimal solution (optimality gap $\approx$ 0%) for all the conducted experiments, except for one specific run in Analysis 3 (for Mix 2). The converged models presented a total number of variables from 1183 to 1999, binary variables from 1140 to 1220, and a total number of constraints from 1316 to 3936. The only instance that did not converge to optimality (gap $\approx$ 5%) presented a total of 3199 variables, 1340 binary variables, and 7866 constraints[1].

5.1 Analysis 1

In the first analysis, the objective is to balance the time of the operators using the equivalent mix and takt time of 6 months, presented in Table 3. It started by defining the objective function (OF), using different variables focusing on different indicators. The first alternative is to minimize the cycle time (CT). The second is to maximize the maximum difference between the station times and the average station time, therefore performing a vertical balancing (MDS). The third option is to minimize the maximum difference between the station time for each model and the average time of the station for each model, thus performing a horizontal balancing ($MDSM_m$). The last alternative is considering the three variables to get a result that encompasses all the three factors, as per Equation (7). The results are compared in Table 8, which presents the cycle time (CT), line efficiency (LE), idle time (IT), balancing delay (BD), smoothness index (SI), and horizontal smoothness index (HSI)[2]. Bold indicates the best obtained value for the considered indicator.

As expected, minimizing the cycle time provides better results for the cycle time and the indicators that depend on it directly: line efficiency, idle time, and balancing delay. Although the smoothness index is influenced by CT and had a good result, it is smaller when minimizing

[1] This instance did not evolve significantly when the execution time was increased up to four hours, so it was decided to maintain the execution time of one hour for all tested instances.

[2] Section 2.3 mathematically defines LE, IT, BD, SI, and HSI.

Table 8 – Comparison of several indicators using different objective functions for Analysis 1

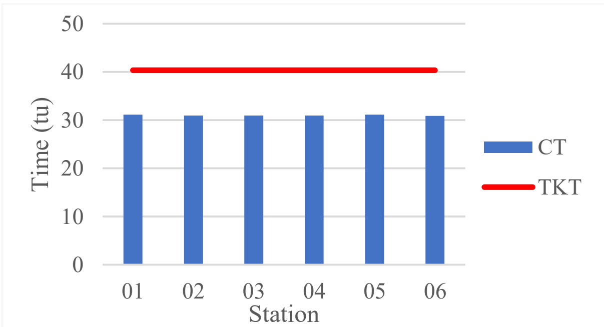
OF Minimize	CT (tu)	LE (%)	IT (tu)	BD (%)	SI	HSI
CT	**31.00**	**99.97**	**0.05**	**0.03**	0.04	5.98
MDS	**31.00**	**99.97**	**0.05**	**0.03**	**0.03**	6.02
$MDSM_m$	32.91	94.19	11.47	5.81	5.97	**3.64**
$C_{CT} \cdot CT + C_{MDS} \cdot MDS + C_{MDSM_m} \cdot \sum_{m=1}^{M} MDSM_m$	31.11	99.64	0.67	0.36	0.34	3.66

Source: Own authorship (2023).

MDS. While minimizing $MDSM_m$, it returned the worst result for cycle time, line efficiency, idle time, balancing delay, and smoothness index but the best for the horizontal smoothness index. The fourth option used all three variables for the objective function, with the coefficients C_{CT}=100, C_{MDS}=10, and C_{MDSM_m}=10, which were obtained after preliminary tests aiming to find a tradeoff between the different goals. Recommendations about these parameters are indicated in definition of Equation (7). Even though the results for CT are only 0.3% worse than options 1 and 2, it is compensated by better horizontal balancing, an essential indicator since the line operates with different models simultaneously, that is why the fourth option is chosen as the objective function.

The obtained result involves a total of 36 task changes, which represents 19% of the tasks changing from one station to another in relation to the initial balancing condition. The line expert validated all the suggested changes as viable. The workload distribution among the stations can be seen in Figure 16.

Figure 16 – Cycle time of line stations for Analysis 1



Source: Own authorship (2023).

As can be seen in Figure 16, the cycle time is below the (average) takt time, which indicates that the balance of activities obtained can meet, in principle, the demand, considering the equivalent mix and the weighted time of the given mix. Compared to the initial situation, which had a CT of 34.62 tu, that is a 10.1% improvement.

The next step is to check if the result would allow meeting the demand during different specific periods by using the same task allocation (balancing solution), but then applying the eight different mixes of production. When the partial demand changes, a new mix is defined, and the weighted time of tasks also changes, affecting the balancing and, consequently, the obtained cycle time. Therefore, it is necessary to validate if the obtained cycle time (CT) respects the takt time (TKT) for all specific mixes, given the balancing determined. The highest station time of each period, that is, the bottleneck, is summarized in Table 9. The results can be seen in Figure 17. As indicated in Table 9, one out of the eight mixes analyzed returns a cycle time higher than the takt time (Mix 2), which means that the proposed balancing would fail to produce the required volume in that specific period. Thus, for the sake of obtaining a more robust approach, Analysis 2 is proposed, and a second study is carried out, as described in Section 5.2.

Table 9 – Summary of takt time and cycle time for different mixes for Analysis 1

Mix	TKT (tu)	CT (tu)
Mix 1	31.83	31.15
Mix 2	31.83	**31.89**
Mix 3	31.83	30.58
Mix 4	31.83	30.73
Mix 5	51.16	36.74
Mix 6	59.46	34.03
Mix 7	44.61	31.44
Mix 8	33.08	30.21

Source: Own authorship (2023).

5.2 Analysis 2

Taking into account that Analysis 1 had one mix with the CT over the TKT and could not meet the demand, in Analysis 2 eight different balances are performed to ensure that the result would allow it to meet the demand for each mix of production. The same objective function and coefficients employed in the last part of Analysis 1 are used, which considered the CT, MDS, and $MDSM_m$ as per Equation (7). The obtained results can be seen in Figure 18.

The cycle time is summarized in Table 10. This is the reference value for balancing, which means this is the best result achievable for cycle time per mix of production. A comparison between Analysis 1 and Analysis 2 results is seen in Figure 19.

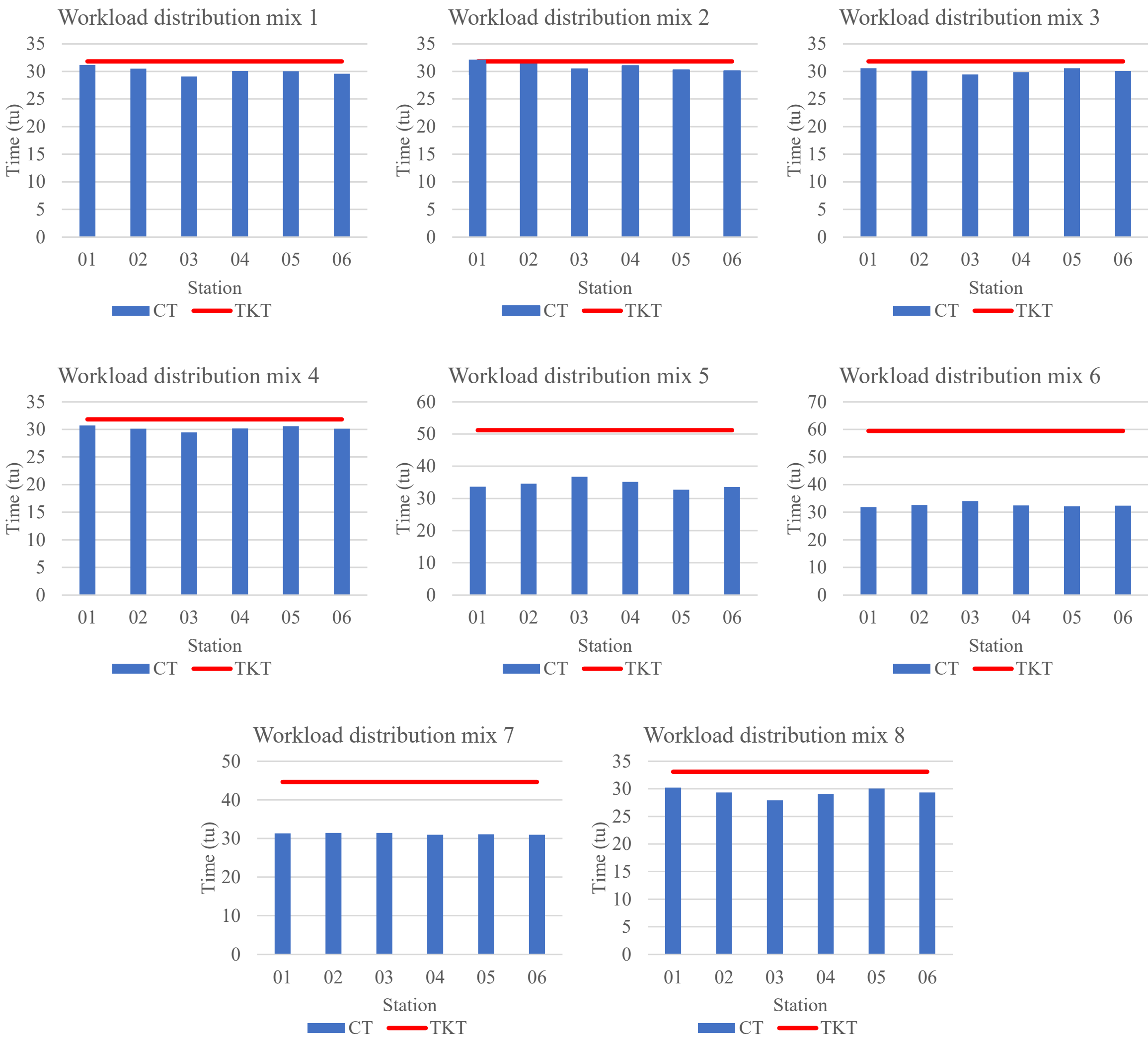
Table 10 – Summary of takt time and cycle time for the different mixes for Analysis 2

Mix	TKT (tu)	CT (tu)
Mix 1	31.83	30.23
Mix 2	31.83	30.97
Mix 3	31.83	30.14
Mix 4	31.83	30.32
Mix 5	51.16	34.67
Mix 6	59.46	32.67
Mix 7	44.61	31.24
Mix 8	33.08	29.43

Source: Own authorship (2023).

As expected, when performing the balancing for each specific period, the result is better, showing a dominance for the result from Analysis 2 compared to Analysis 1. In this case, all obtained cycle time met, with quite a margin, the takt time. What is noticed is that there are many changes in tasks between each period, as per Table 11. This means that period 1 had to

Figure 17 – Workload for each period considering task allocation obtained previously on Analysis 1



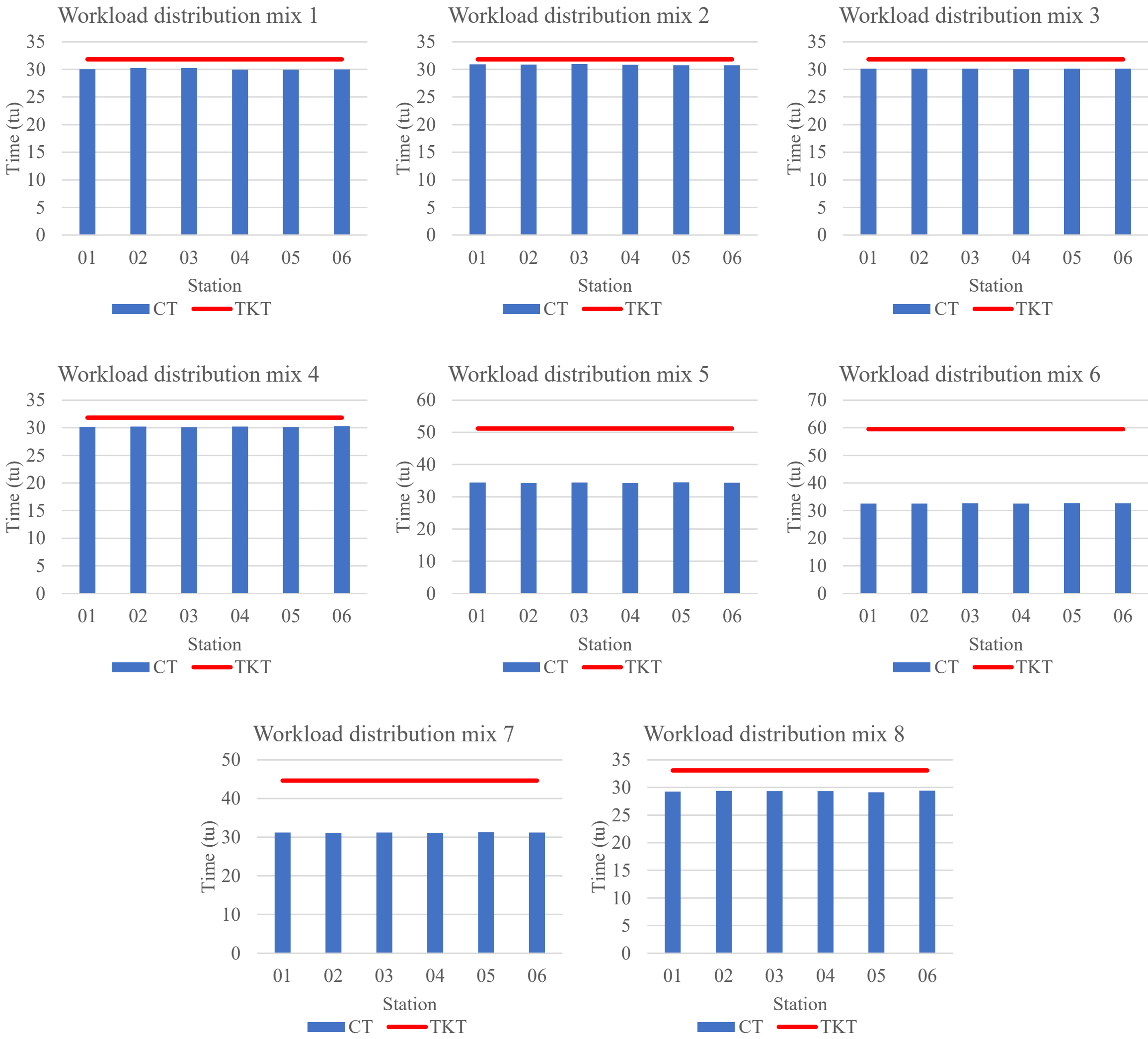
Source: Own authorship (2023).

change the allocation of 36 tasks compared to the initial allocation; period 2 had 35 other task changes compared to period 1, and so forth.

Many changes mean investments in pieces of equipment and tools on the line and time for training the operators. Sometimes, the production mix duration will not even be long enough to implement all the necessary changes. To avoid that, a variable is added, as indicated in Equation (23), to aid in restricting the number of changes. The objective function is then updated to Equation (24). To choose the coefficient for the WCh variable, tests are performed for the C_{WCh} weight coefficient from 0 to 100, as seen in Figure 20.

In particular, NCh indicates the number of changes in relation to the allocation of tasks to stations established in Proposal Analysis 2 (initial line condition for the eight analyzed mixes). NCh is calculated in the post-processing of the mathematical model, considering the initial allocation of tasks: $NCh = \sum_{t=1}^{T} \sum_{s=1 \mid xI_{t,s}=1}^{S} (1 - x_{t,s})$. As the coefficient increases, the CT also increases; therefore, choosing a coefficient that limits the number of task changes without

Figure 18 – Workload for each period considering task allocation obtained previously on Analysis 2 without considering the number of changes

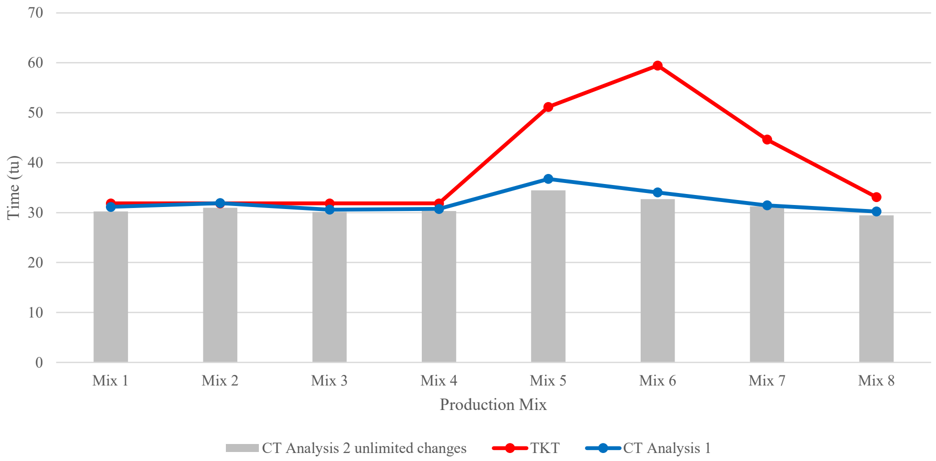


Source: Own authorship (2023).

deteriorating the CT too much is crucial. As it can be seen in Figure 20, the curves of NCh and WCh follow a similar behavior, and even though the objective function considers the WCh, the result analysis takes into account the number of changes in NCh, to facilitate the understanding and comparison.

The coefficient C_{WCh} chosen is five because it allowed a reasonable number of changes in practical terms without compromising the CT too much, a choice that was corroborated by the specialists from the company. Since the chosen weight coefficient is way lower than the coefficient of CT, the primary objective is still to minimize the cycle time, but now, minimum gains of time will not come with the cost of multiple changes. For this study, each task received a weight of change, using 1 for simple changes that did not affect the logistical supply or tools and short duration of operator training, 3 for changes with medium complexity that have changes in the logistical supply, and a moderate duration of the training, and 5 for highly complex changes, with tooling changes, high investment costs and extended training for operators.

Figure 19 – Comparison between takt time, results from Analysis 1 and Analysis 2 without limiting changes



Source: Own authorship (2023).

Table 11 – Number of tasks changes compared to the last period for Analysis 2 without limiting the number of changes

Period	Number of tasks changes
Initial	-
Period 1	36
Period 2	35
Period 3	34
Period 4	35
Period 5	20
Period 6	21
Period 7	17
Period 8	36

Source: Own authorship (2023).

The model is executed using Mix 1, and a coefficient K equal to one is initially used.[3] The result, called Proposal Analysis 2, can be seen in Table 12 with a comparison with the initial allocation.

Table 12 – Comparison between cycle time of initial situation and Proposal Analysis 2

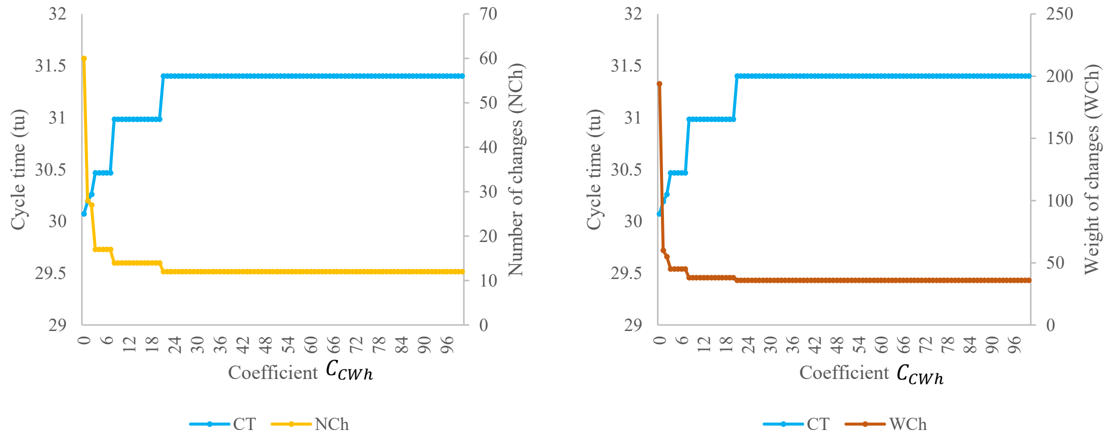
Mix	TKT (tu)	CT (tu)	NCh
Initial	31.83	34.62	-
Proposal Analysis 2	31.83	30.47	17

Source: Own authorship (2023).

Proposal Analysis 2 allowed to reach the takt time, a fact that did not occur in the initial situation (according to Figure 12), and brought a reduction of 12.0% in cycle time. For the optimized solution, a total of 17 changes were proposed, with an implementation cost of approximately $50,000.00 monetary units (Table 13). The feasibility of the obtained solution is validated by the line specialist by observing the changes in operations and confirming if all the line and product restrictions are respected. The list of tasks changes are shown on Table 13. Only two tasks (37

[3] Mix 1 was adopted as a reference based on recommendations from the company specialists. For future works, other mixes can be tested as the reference one to analyze the obtained results.

Figure 20 – Behavior of cycle time CT, number of changes NCh, and weight of changes WCh according to the coefficient C_{WCh}



Source: Own authorship (2023).

and 72) demanded monetary investments to purchase two new screwdrivers. The other 15 tasks required only updating the work instructions and training the operators with the new assignment of activities; therefore, no costs of training were considered by the company. The values associated with the changes met the profitability criteria with a large margin, with a payback of 0.2 years. The workload distribution of the proposed optimized solution can be seen in Figure 21.

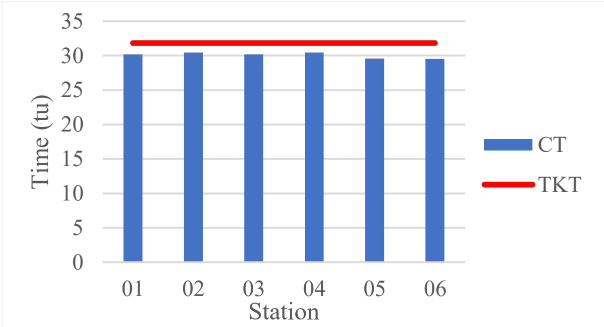
Table 13 – Tasks changes between initial situation and Proposal Analysis 2

Task	Description	Current Station	Proposed Station	Action Plan	Investment ($)
36	Assembly of the frontal right door on BIW	1	3	Update work instruction	-
37	Fastening of the frontal right door on BIW	1	3	Purchase new screwdriver	25,000
41	Assembly of AC structure	4	6	Update work instruction	-
71	Assembly of the frontal left door on BIW	2	4	Update work instruction	-
72	Fastening of the frontal left door on BIW	2	4	Purchase new screwdriver	25,000
73	Inspection of BIW	2	4	Update work instruction	-
74	Manually fastening of right side A-pillar screw	3	5	Update work instruction	-
108	A-pillar rework	3	5	Update work instruction	-
118	Assembly of two nut plates on the right side	3	5	Update work instruction	-
124	Manually fastening of left side A-pillar screw	4	6	Update work instruction	-
125	Review the condition of the frontal right door	4	2	Update work instruction	-
126	Review the presence of pointed spots on frontal right door	4	2	Update work instruction	-
130	Assembly of tailgate on BIW	4	5	Update work instruction	-
131	Assembly of tailgate six screws	4	5	Update work instruction	-
132	Assembly of rotule on BIW	4	6	Update work instruction	-
133	Assembly of two nut plates on the left side	4	6	Update work instruction	-
157	Lock tailgate support	5	6	Update work instruction	-

Source: Own authorship (2023).

Figure 22 compares the cycle times specified for each model for the current situation and Proposal Analysis 2. Note that the time of some models exceeds the takt time. However, this can be circumvented (or at least minimized) with proper sequencing, as in the works carried out by Meira (2015), Lopes *et al.* (2018), Lopes *et al.* (2020a). The proposed solution, however,

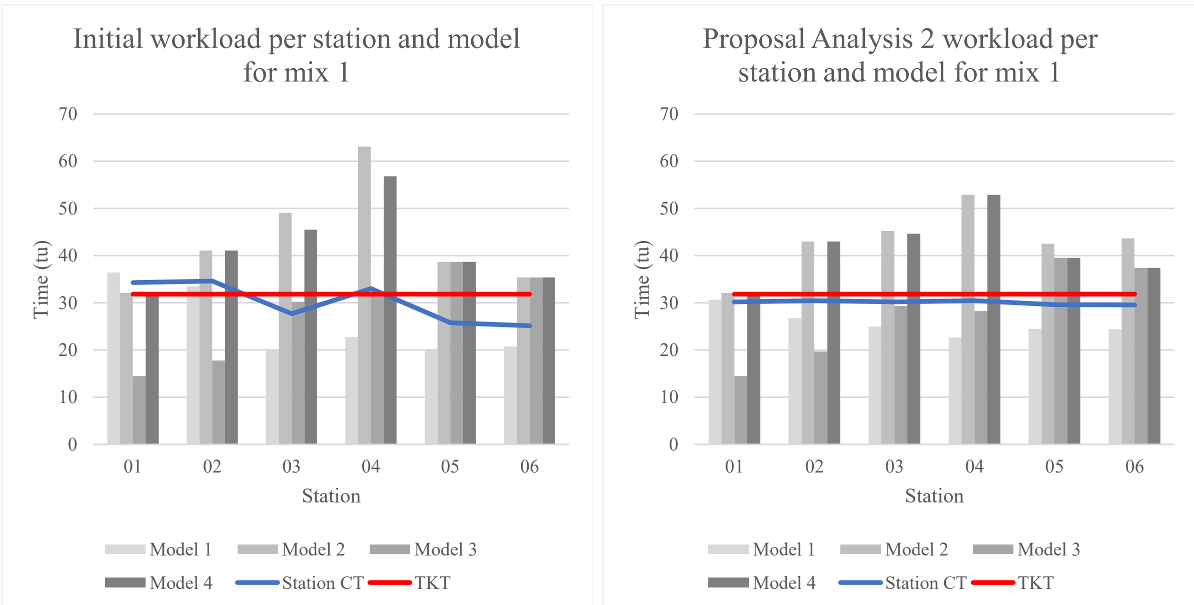
Figure 21 – Cycle time of line stations for Proposal Analysis 2 for mix 1



Source: Own authorship (2023).

improves the horizontal smoothness index by 14.1% in relation to the current proposal, seeking to reduce the time differences between different models at stations.

Figure 22 – Comparison between the initial allocation and Proposal Analysis 2 load distribution for the different models



Source: Own authorship (2023).

Once the result (Proposal Analysis 2) meets the demand, the coefficient K of the objective function is changed to a value equal to 10, which proved to be sufficient to avoid unwanted changes in relation to the distribution of tasks established in Proposal Analysis 2. The model is run for the seven other production mixes, covering a period of six months between 2021 and 2022, and its results are shown in Figure 23. Table 14 summarizes the obtained cycle times.

As can be seen from Table 14, for all cases, the cycle time is lower than the takt time available to the mix, which means it is possible to meet customer demands. The fact that all the analyzed scenarios do not present changes in activity indicates that Proposal Analysis 2 tends to meet the needs of partial demand changes occurring in the six months, in which demand varies significantly. A comparison involving the current situation, Proposal Analysis 2, and production mix indicators 2 to 8 can be found in Figure 24. Even though the cycle time for all eight periods is higher when minimizing the number of changes, the difference is considered acceptable because the most crucial goal is to be below the takt time, and the obtained solution would only require one line balancing, investment, and training of operators.

Table 14 – CT results and number of changes of mix 2 through 8 compared to the previous mix

Mix	TKT (tu)	CT (tu)	NCh
Initial	31.83	34.62	-
Mix 1	31.83	30.47	17
Mix 2	31.83	31.81	0
Mix 3	31.83	30.75	0
Mix 4	31.83	30.75	0
Mix 5	51.16	38.02	0
Mix 6	59.46	35.28	0
Mix 7	44.61	32.75	0
Mix 8	33.08	29.44	0

Source: Own authorship (2023).

As indicated in Figure 24, the cycle time is lower than the takt time in all scenarios, with a more significant difference in mixes 5, 6, and 7, which had a higher takt time (thus a lower demand) than the others, according to Figure 24a, in addition to a higher percentage of the production demand of models 2 and 4, which have a higher workload. Such factors explain the other curves in Figure 24b-f, since, according to Equation 1, the minimum number of operators could be less than six (five operators for mix 5 and 7, and four operators for mix 6), so there is a drop in efficiency for those three mixes, which also leads to higher total idleness, balance delay, and smoothness index. It was decided not to change the number of operators for the balancing but instead reduce the hours worked during the week. This decision is aligned with the procedures adopted by the company; for future research, the worker's reduction can be evaluated. Even so, all scenarios' indicators show improvements compared to the current situation. Table 15 compares the results between CT taking and not taking into account the number of changes.

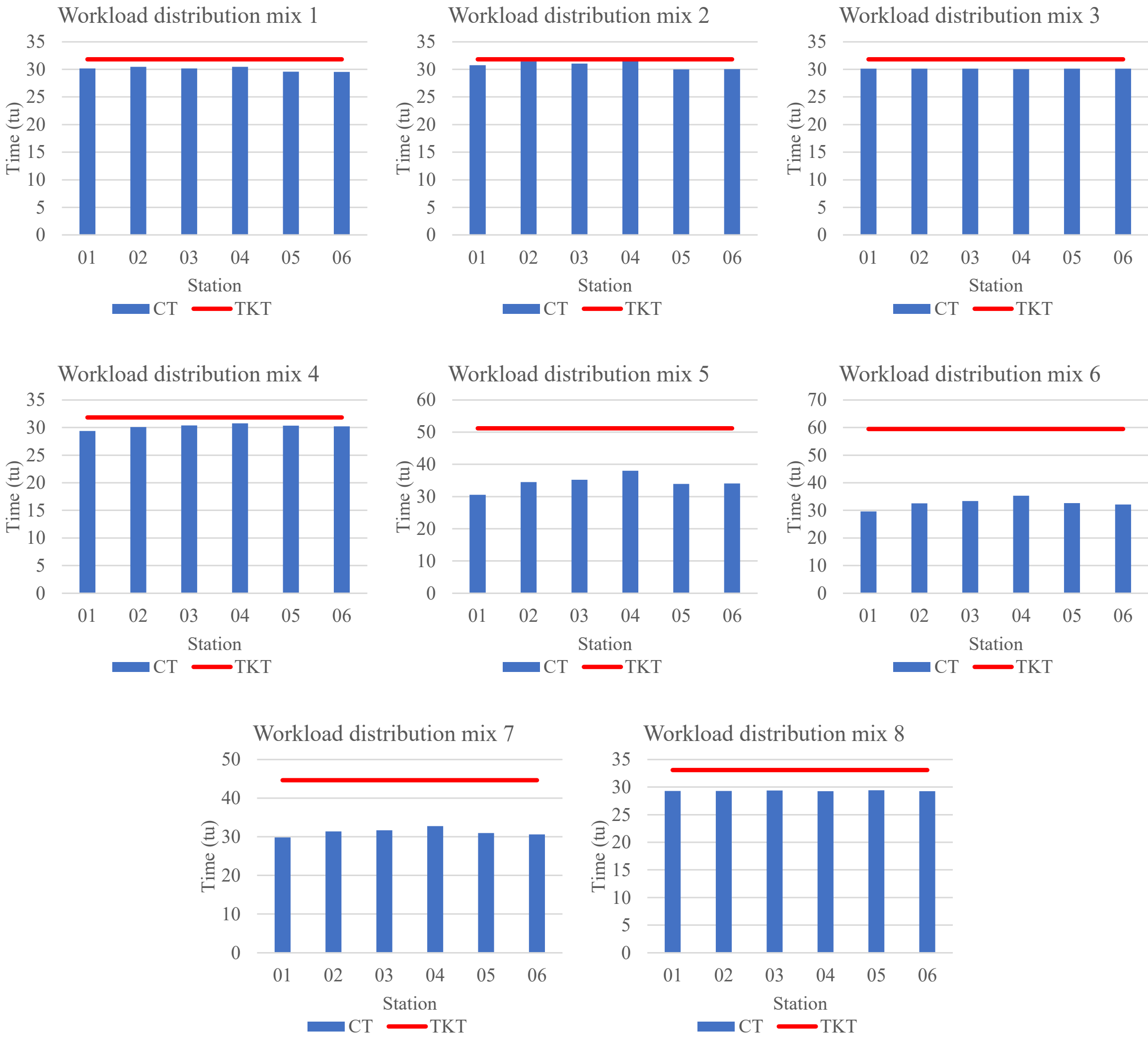
Table 15 – Comparison between the CT without minimizing NCh and minimizing NCh

Mix	**CT without minimizing NCh** (tu)	**CT minimizing NCh** (tu)	**CT increase when minimizing NCh**
Mix 1	30.23	30.47	0.8%
Mix 2	30.97	31.81	2.7%
Mix 3	30.14	30.75	2.0%
Mix 4	30.32	30.75	1.4%
Mix 5	34.67	38.02	10.3%
Mix 6	32.67	35.28	8.0%
Mix 7	31.24	32.75	4.8%
Mix 8	29.43	29.44	0.0%

Source: Own authorship (2023).

As highlighted before, adding the restriction of minimizing the number of changes implies that the result of CT will be worse. However, the result is still better than the TKT. Proposal Analysis 2 proved to be "resilient" to different customer demands, a feat that the line specialist considers preferable to having a smaller CT requiring frequent changes.

Figure 23 – Workload for each period considering task allocation obtained previously on Analysis 2 and considering the number of changes



Source: Own authorship (2023).

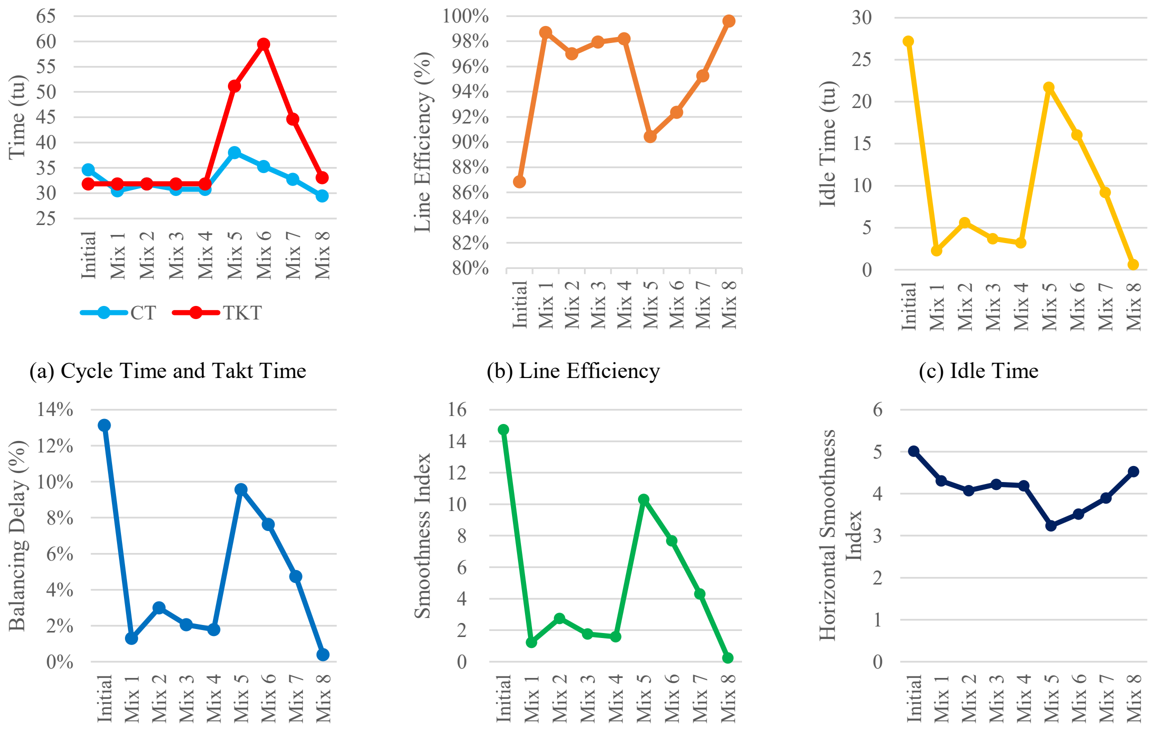
This solution of task allocation was analyzed by the line specialist and proved to be feasible, so it was implemented on the real assembly line, improving the indicators described previously.

Since it is shown in the Figure 22 that some models had a processing time higher than the TKT, Analysis 3 considers in an integrated way the sequencing and the balancing of different models to improve this situation.

5.3 Analysis 3

This section highlights the main results obtained by the proposed balancing and sequencing approach presented in Subsection 4.1.3. The goal is to verify the results of a model that would represent a theoretical problem inspired by the real one. The production mix used is Mix 1,

Figure 24 – Comparison between indicators for: initial situation; Proposal Analysis 2 (mix 1); and mix 2 to mix 8



(a) Cycle Time and Takt Time (b) Line Efficiency (c) Idle Time

(d) Balancing Delay (e) Smoothness Index (f) Horizontal Smoothness Index

Source: Own authorship (2023).

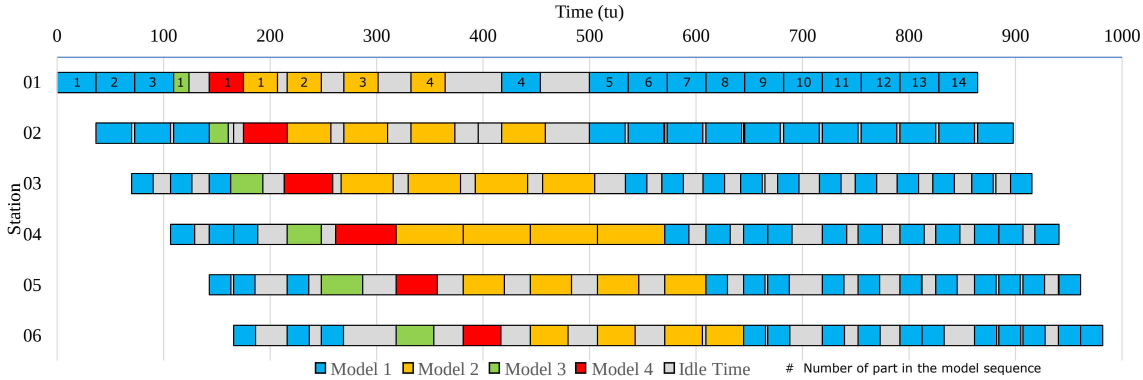
which has 70% of demand for model 1, 20% for model 2, 5% for model 3, 5% for model 4, and a MPS of 20 parts. The objective function is to minimize the $CTST$ as per equation (38).

5.3.1 Sequencing of initial allocation

The first step is to sequence the initial allocation of tasks to evidence the best result for the steady-state cycle time of the initial allocation. This information is later used to compare with other solutions. Notice that the balancing for the initial allocation is fixed, and just the sequencing is performed. Figure 25 brings the Gantt chart, where the Y-axis represents stations 1 through 6, and the X-axis represents the time, in time units (tu). Each box represents a product, according to the result of the sequencing of the four different models produced between the 20 parts. Idle times are also indicated in the Gantt chart. To facilitate understanding, the first row has the sequence number of the part, which remains the same in the six stations.

The sequence indicated in Figure 25 would be of three cars of Model 1, followed by one car of Model 3, then one Model 4, soon four Model 2, and finally another 11 Model 1. The sequencing brought a $CTST$ of 43.22 tu. Compared with the CT of 34.62 tu, the result is 24.8% higher. Thus, idle times are indeed influencing in a negative way the line productivity.

Figure 25 – Gantt chart for the sequencing of the initial allocation

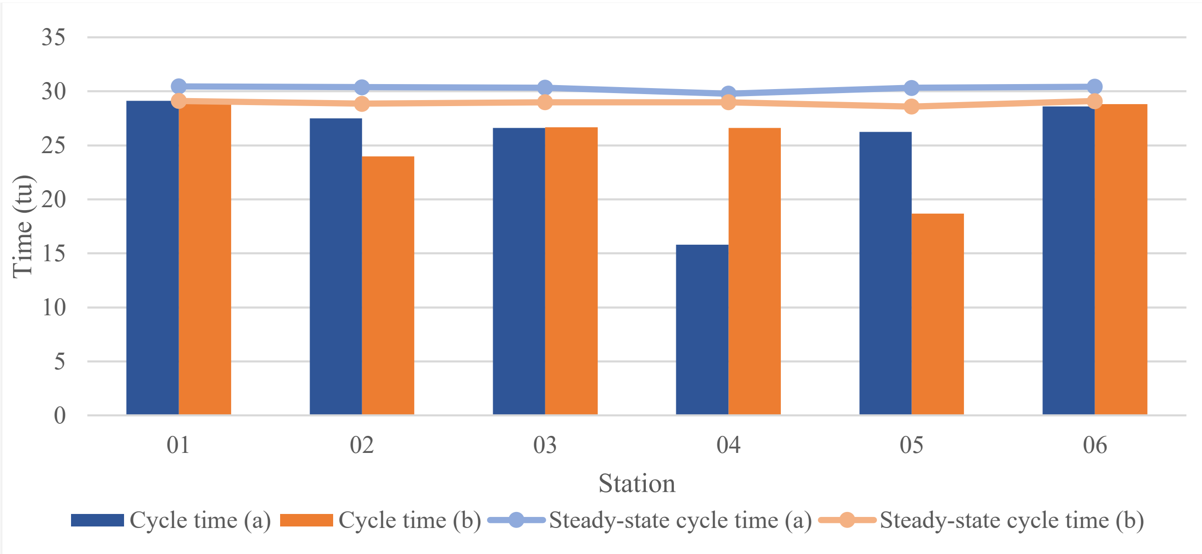


Source: Own authorship (2023).

5.3.2 Validation of sequencing model

In order to validate this significant difference between CT and $CTST$, an experiment is conducted. A test is performed using only one model (Model 1), first minimizing the CT (alternative a) and next minimizing the $CTST$ (alternative b). The results can be seen in Figure 26 and Figure 27.

Figure 26 – Comparison between cycle time and steady-state cycle time for the test with one model

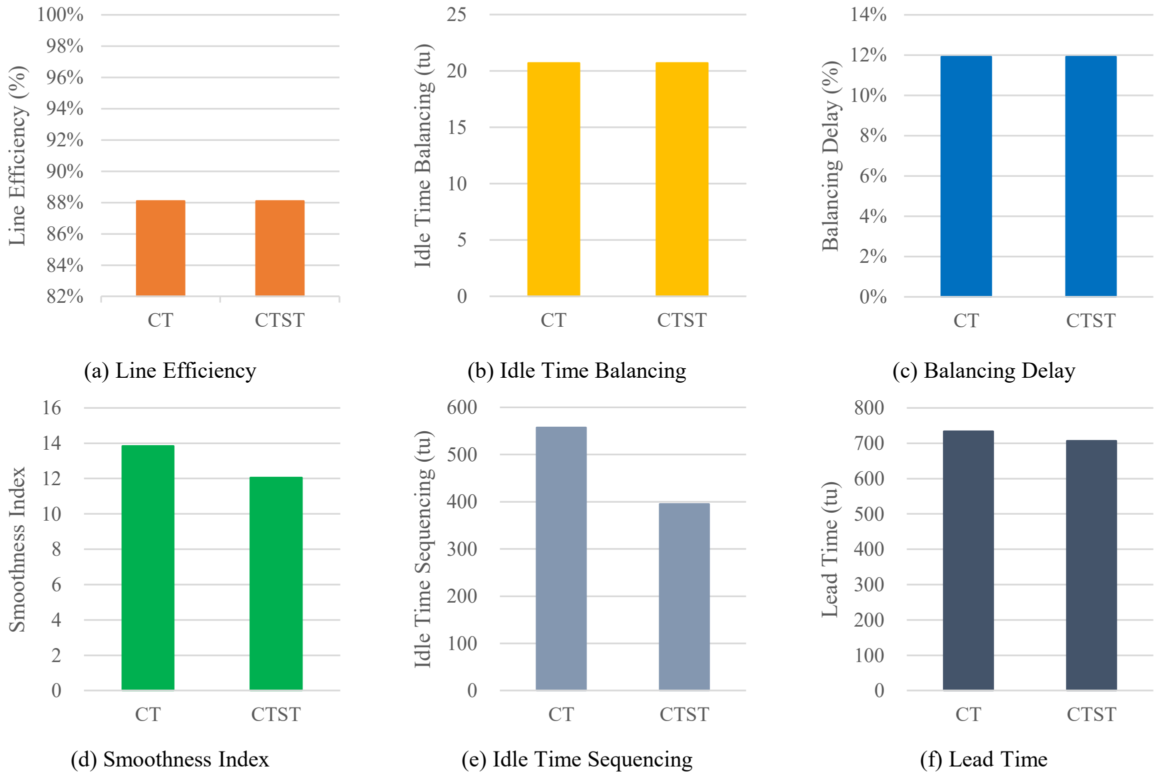


Source: Own authorship (2023).

As indicated in Figure 26, the bottleneck-station considering just the balancing, that is, the pure processing time of products, is 29.11 tu, for both alternatives. However, when considering the sequencing approach, alternative (b) proved to dominate alternative (a) with relatively smaller steady-state cycle time values.

As can be seen in Figure 26 and Figure 27, minimizing the $CTST$ with only one model presents the same cycle time as minimizing the CT; notwithstanding, the opposite is not valid when considering sequencing aspects. When comparing the balancing indicators (line efficiency, idle time, balancing delay, smoothness index), the results for $CTST$ are rather similar to CT (un-

Figure 27 – Indicators for the test with one model



(a) Line Efficiency (b) Idle Time Balancing (c) Balancing Delay (d) Smoothness Index (e) Idle Time Sequencing (f) Lead Time

Source: Own authorship (2023).

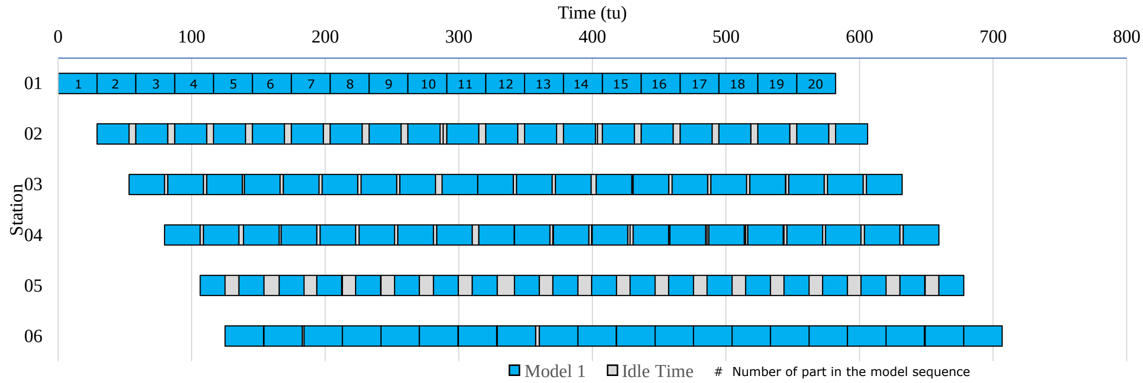
less for balancing delay; this might be because there could be multiples solutions with the same CT, but with different workload for other stations). Meanwhile, when considering the indicators for sequencing (lead time and idle time for sequencing - the latter is the sum of all starvations and blockages in all stations and parts), the results are better for the $CTST$ minimization. Since the balancing indicators essentially consider the cycle time (weighted by the partial demand), they are theoretical values that are not necessarily appropriate when reviewing sequencing aspects.

Figure 28 brings the Gantt considering only one model and minimizing the $CTST$. The first station, where there is no idle time, has both the CT and $CTST$ equal to 29.11 tu. Thus, it is noted that while working with only one model, the steady-state cycle time tends to be equal to the cycle time of just balancing. Whereas, the difference between CT and $CTST$ for mixed-model is due to the difference in workload between products, their mix, and the particular restrictions of this problem.

5.3.3 Sequencing of Proposal Analysis 2

Once the sequencing part of the model is validated, the next step is to sequence the production for the task allocation obtained for the balancing from Proposal Analysis 2. The obtained result can be seen in the Gantt chart of Figure 29, admitting a sequence of 13 products

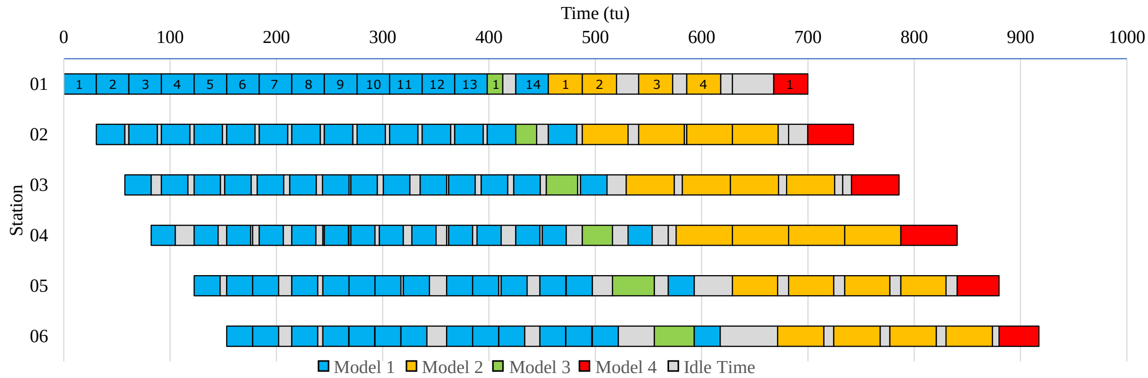
Figure 28 – Gantt chart for the test with one model minimizing $CTST$



Source: Own authorship (2023).

of Model 1, one product of Model 3, followed by one more of Model 1, then four of Model 2, and finally one of Model 4, resulting in a a $CTST$ of 38.20 tu against 30.47 tu of CT.

Figure 29 – Gantt chart for the sequencing of the Proposal Analysis 2



Source: Own authorship (2023).

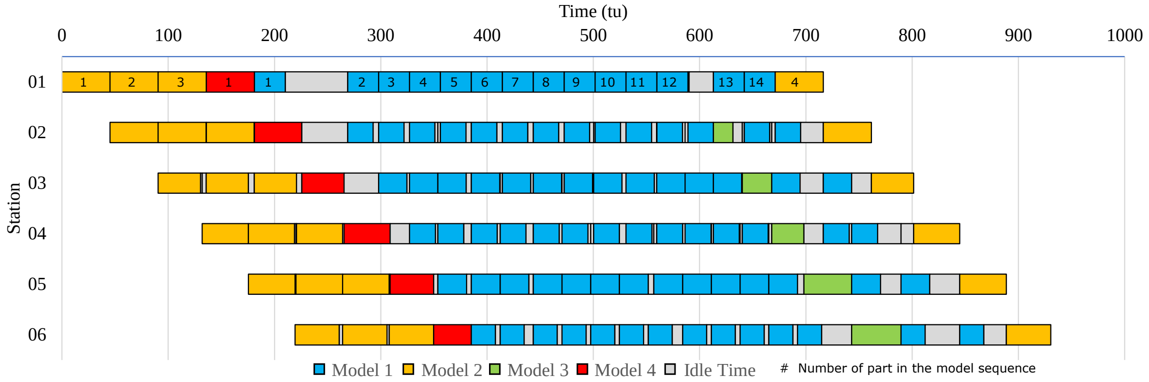
5.3.4 Integrated balancing and sequencing

Finally, the model considering both the balancing and the sequencing is run. In this case, balancing and sequencing aspects are considered to get the best outcome possible for $CTST$. The Gantt is shown in Figure 30.

In Figure 30, the sequence starts with three parts of model 2, then one model 4, followed by 12 of model 1, then one model 3 (almost not visible in station 1 due to the relative duration), two of model 1 again, and finally one model 2, resulting in a $CTST$ of 35.82 tu and 40 tasks changes. An interesting finding is that the workloads for the models in stations had a significant difference between them, as evidenced in Figure 31. Model 3 had only 1.0 tu for the first station and 46.4 tu for the sixth station, representing a difference of 98%. The results are compiled in Table 16.

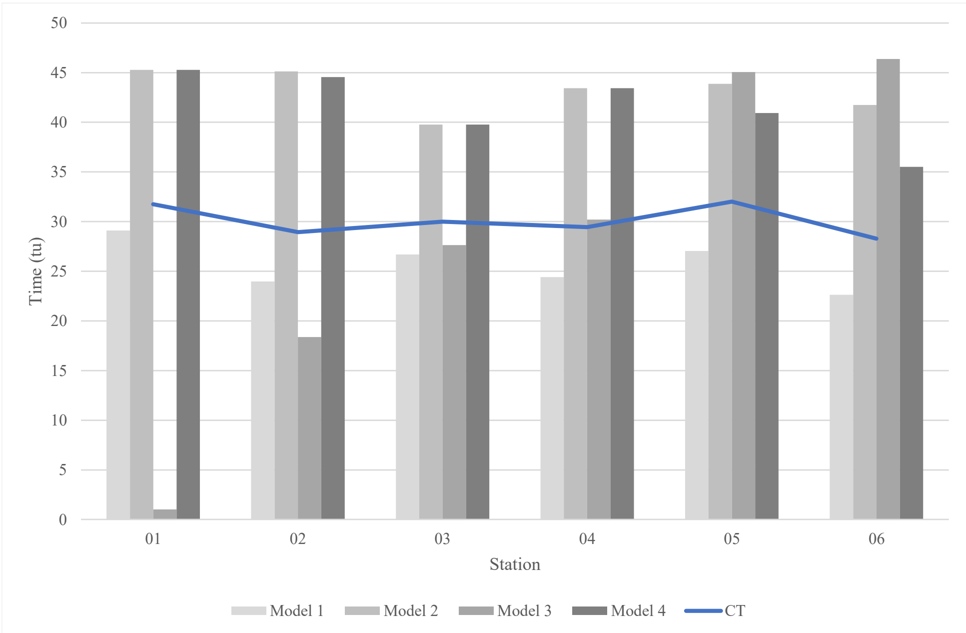
Although the CT is higher for the balancing and sequencing in relation to Proposal Analysis 2, it is preferable to minimize the $CTST$ once the CT does not consider the idle time originated from starvations or blockages between stations, being purely the time of processing the

Figure 30 – Gantt chart for the balancing and sequencing of mix 1, without considering the number of changes of tasks in relation to the initial allocation



Source: Own authorship (2023).

Figure 31 – Workload of different models for balancing and sequencing alternative



Source: Own authorship (2023).

Table 16 – $CTST$ comparison for the initial situation, Proposal Analysis 2, and balancing and sequencing done simultaneously

Solution	CT (tu)	$CTST$ (tu)
Sequencing Initial	34.62	43.22
Sequencing Proposal Analysis 2	30.47	38.20
Balancing and Sequencing	32.16	35.82

Source: Own authorship (2023).

parts. Because of that, Analysis 3 hereafter considers only the $CTST$ and not the CT. Again, for this reason, the restriction (14), which indicates that the CT should be equal to or smaller than the TKT, is removed since the approach no longer considers the CT and the $CTST$ has proven to be above the TKT on the theoretical problem considered. Indeed, some additional tests were performed, removing all the precedence restrictions, incompatibilities, fixed tasks, and distance between tasks. For Mix 1, the best $CTST$ possible would be 33.87 tu, but this does not respect any of the line's physical restrictions. Thus, considering the sequencing conditions, the theoretical takt time imposed by the demand requirements (31.83 tu) could not be reached in a sequential arrangement of simple stations. This, in practical terms, means that the productive capacity would be below the expectation of demand planners when considering more rigorously balancing-sequencing aspects, a fact that is evidenced by the proposed approach aid.

Following the methodology of Analysis 3, the model is executed for the eight different mixes, using the objective function centered on minimizing $CTST$, to have the best $CTST$ possible as the reference value. The obtained results are summarized in Table 17, which brings the obtained $CTST$ in tu, the number of necessary changes NCh, and the maximum idle time value $MaxIT$ in tu.

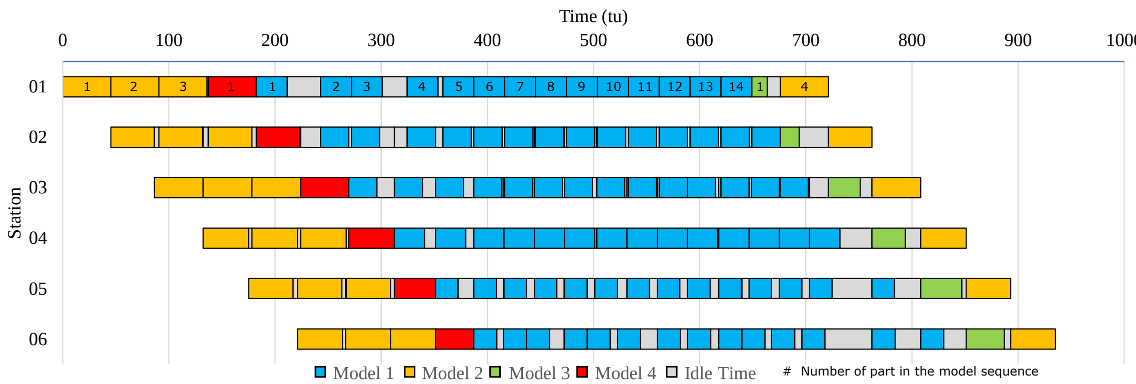
Table 17 – Reference value of $CTST$ for the eight mixes

Mix	$CTST$ (tu)	NCh	$MaxIT$ (tu)
Mix 1	35.82	40	49.0
Mix 2	35.03	18	51.0
Mix 3	35.38	33	45.0
Mix 4	35.38	18	42.0
Mix 5	39.87	43	44.0
Mix 6	40.93	26	43.0
Mix 7	36.06	28	42.0
Mix 8	34.57	3	38.0

Source: Own authorship (2023).

Once again, the best possible $CTST$ results come with the expense of multiple changes for every partial demand update, which may not be operational, as already explained in this study. To solve this issue, the objective function is updated to Equation (39), adding the restriction of minimizing the number of changes. The coefficient C_{CTST} is equal to 100 while the coefficient C_{WCh} is still equal to five. The parameters were obtained after preliminary tests which follow the recommendation given in the definition of Equation (40). The Gantt can be seen in Figure 32. Differently from the previous result, the $CTST$ is 36.10 tu, which is 0.8% higher than the previous result without considering the number of changes.

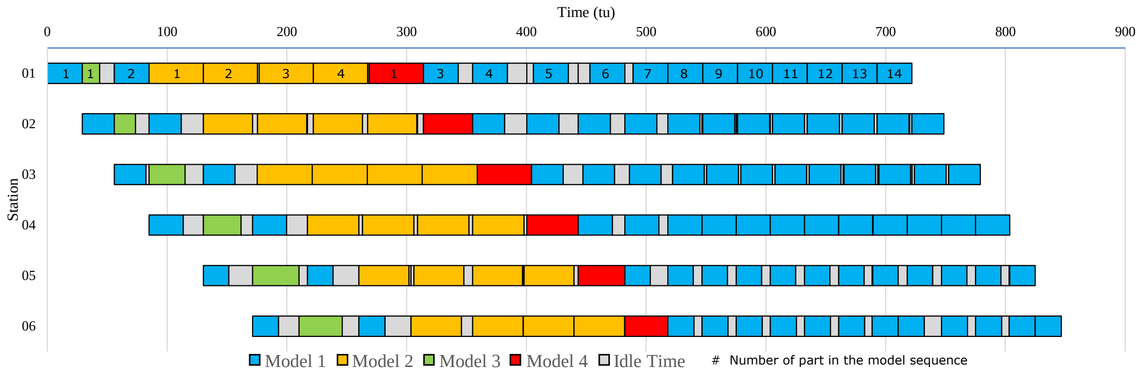
Figure 32 – Gantt chart for the balancing and sequencing of mix 1, considering the number of changes of tasks in relation to the initial allocation



Source: Own authorship (2023).

Besides restricting the number of changes, reducing the variation of idle times between stations is also desired. The objective function is then updated to (40), adding the minimization of the variable $MDSM_m$. It is also added the minimization of MDS to improve the horizontal balancing and the minimization of the $MaxIT$ to seek the balance of the idle times between the parts. The coefficients K, C_{MDSseq}, $C_{MDSMseq}$, and C_{MaxIT} are equal to one, following the order of magnitude defined in Equation (40). The result is presented in Figure 33. The $CTST$ obtained is 36.10 tu, which is the same as the previous objective function used. A total of 11 task changes are necessary and are detailed on Table 18.

Figure 33 – Gantt chart for the balancing and sequencing of mix 1, considering the number of changes of tasks in relation to initial allocation, the $MaxIT$, the MDS, and $MDSM_m$



Source: Own authorship (2023).

Table 18 – Tasks changes between Initial Situation and Proposal Analysis 3

Task	Description	Current Station	Proposed Station	Action Plan	Investment ($)
32	Sanding of truck bed	1	3	Update work instruction	-
36	Assembly of the frontal right door on BIW	1	3	Update work instruction	-
37	Fastening of the frontal right door on BIW	1	3	Purchase new screwdriver	25,000
41	Assembly of AC structure	4	6	Update work instruction	-
71	Assembly of the frontal left door on BIW	2	4	Update work instruction	-
72	Fastening of the frontal left door on BIW	2	4	Purchase new screwdriver	25,000
73	Inspection of BIW	2	4	Update work instruction	-
108	A-pillar rework	3	5	Update work instruction	-
118	Assembly of two nut plates on the right side	3	5	Update work instruction	-
128	Sanding of internal left inner side	4	1	Update work instruction	-
133	Assembly of two nut plates on the left side	4	6	Update work instruction	-

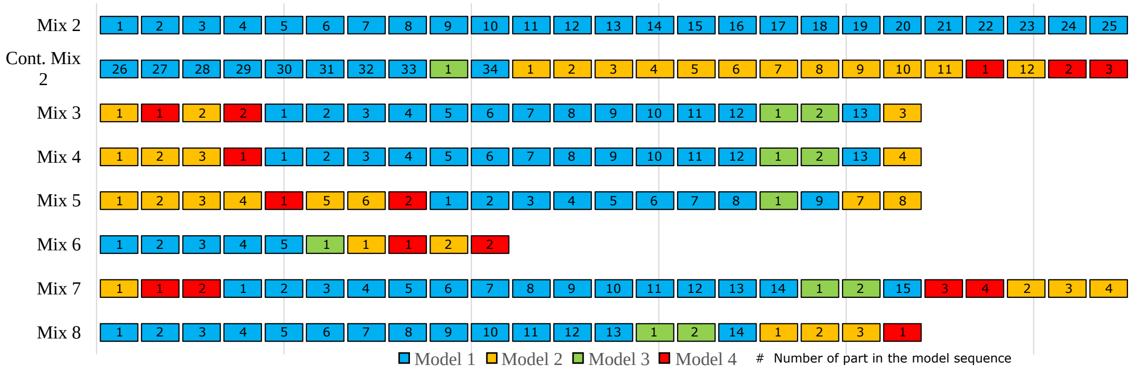
Source: Own authorship (2023).

Even though this was a theoretical analysis, the allocation of tasks obtained, were analyzed by the specialist, and proved to be feasible. The only tasks that required monetary investments were the ones associated with the new screwdrivers; however, these two tasks were already implemented based on Proposal Analysis 2. Thus, there would be no changes besides updating work instructions and the training of operators. Comparing Table 13 and Table 18, the first had 17 changes of tasks while the latter had 11, 9 of them already displayed on Proposal Analysis 2. Only tasks 32 and 128 were different suggestions of changes in the balancing and sequencing proposal. By observing Table 16 it is possible to notice that the CT for the balancing and sequencing (32.16) is higher than the one in Sequencing Proposal Analysis 2 (30.47), which indicates that the number of changes of this balancing-sequencing analysis is smaller than

Proposal Analysis 2; however, the $CTST$ is better for the balancing-sequencing, because the freedom of balancing and sequencing allowed to find a sequence of products that suits better the problem.

Since adding more restrictions did not affect the main goal, which is to minimize the $CTST$, the model is then executed for the other seven mixes, but now with the weight value of 10 to limit the number of changes. Thus, the coefficient K is changed from 1 to 10, and the model is executed for mixes 2 through 8. The sequence obtained for each mix is presented on Figure 34. It is possible to notice that Mix 2 has an MPS of 50 with 34 products of model 1, 12 of model 2, one of model 3, and three of model 4. The remaining mixes can be understood in an analogous way with MPS values of: 20 for Mix 3, Mix 4, Mix 5, and Mix 8; 10 for Mix 6; and, 25 for Mix 7. Mix 1 can be seen on Figure 33.

Figure 34 – Schematic representation of sequences of mix 2 to mix 8



Source: Own authorship (2023).

Table 19 presents the obtained result of the number of changes and the maximum idle time for each one of the mixes. According to Table 19, the $CTST$ obtained when using K=10 in Equation (40) increased when compared to the previous solution, which used K=1 (Table 17). However, the number of changes from one period to another decreased significantly, thus, having balanced for mid/long-term and having different sequences of products according to the demand of the customer, which is a short-term decision. The maximum idle time between stations is also improved in relation to Table 17, meaning the idle times would be more distributed throughout the productive line.

Table 19 – Results of $CTST$, number of changes, and maximum idle time for all mixes

Mix	$CTST$ (tu)	NCh	$MaxIT$ (tu)
Mix 1	36.10	11	21.7
Mix 2	35.25	0	21.7
Mix 3	35.98	0	21.7
Mix 4	36.01	0	21.7
Mix 5	40.27	0	21.7
Mix 6	41.52	0	27.5
Mix 7	36.58	0	21.7
Mix 8	35.17	0	21.7

Source: Own authorship (2023).

The obtained results considering balancing-sequencing aspects in an integrated manner are of fundamental importance to provide a better understanding of the real potentials of the line when facing a mixed-model productive environment.

Based on the obtained results, Chapter 6 presents the main conclusions from the developed work and indicates future directions of research.

6 CONCLUSION

This study presents the resolution of a balancing and sequencing problem of an assembly line in a car factory. The proposed solution approach is based on the development of mathematical models, in specific Mixed Integer Linear Programming models. Part of the study (Analysis 1 and Analysis 2) involved an *in loco* experience in a real-world productive line, including data collection, validation of proposed solutions, and implementation of obtained answers in the line. Afterward, a more theoretical study was developed (Analysis 3), but inspired by the real line.

Chapter 1 briefly introduced the subject of balancing and sequencing, the general and specific objectives, as well as the justification of the study. Chapter 2 discussed the related concepts to line balancing and sequencing, which are necessary for the project's development. Chapter 3 described the real-world problem by presenting different models, work contents, and the precedence diagram.

Chapter 4 presented the proposed methodology to solve the balancing and sequencing problem, with the three analyses developed and the mathematical model for each of them: Analysis 1 considered a general balance for the six months of production; Analysis 2 initially took into account a balance for each production mix and then added a factor to limit the number of changes, thus generating a proposal that is resilient to changes in customer demand; and finally, Analysis 3, simultaneously balanced and sequenced a theoretical problem based on the real one. Analysis 3, involves finding a proposal that met different production demands of vehicles, in addition to validating the balancing-sequencing approach created, which encompasses the concept of CT (Cycle Time) and $CTST$ (Steady-State Cycle Time).

Chapter 5 shared the results obtained in each analysis context. Analysis 1 showed an improvement of 10.1% in cycle time with respect to the practical observed condition. However, it failed to meet the takt time for a specific period. Analysis 2 (in part A) balanced each production period separately. Although it improved CT, it suggested many task changes, which are associated with investments in equipment, line modifications, and operator training, which resulted in expenses. For this reason, a variable was added in order to minimize the number of changes, and Proposal Analysis 2 was generated, which demonstrated an improvement of 12.0% of the cycle time and proved capable of satisfying another seven production demand scenarios. After validations by the line specialist, this proposal was implemented on the line.

Analysis 3 finally carried out a theoretical balancing and sequencing of production, initially calculating the $CTST$ of the balancing obtained in Analysis 1 (43.22 tu) and in Analysis 2 (38.20 tu). These results were compared with the balancing and sequencing done simultaneously (35.82 tu), which brought an improvement of 17.1% compared to the initial $CTST$. The theoretical allocation of tasks and sequence of products were analyzed by the specialist and considered valid for the layout under consideration. With the results of Analysis 3 it was found that the balancing indicators are not fully adequate to analyze results where sequencing aspects have also to be considered, because these indicators do not properly consider blockage and starvation. Indeed, they consider an equivalent model whose only idle time is due to the average time of the station, which takes in account the partial demands of each product. Then, the balancing-sequencing model was executed for the different production mixes, where many changes were found between the periods. Due to this factor, a solution was sought to meet the different demands through the addition of the variable that restricts the number of them, having a $CTST$ of 36.10 tu for Mix 1 with 11 task changes, which resulted in an improvement of 16.5% compared to the initial allocation. Then, this obtained solution was applied to the following seven mixes without changing tasks, which allowed to have only one event of changes on the line and then worked on the short-term decisions with the sequence of products.

As evidenced by the developments and obtained results, the general and the specific objectives initially proposed in Chapter 1 were addressed. As a summary, balancing the line was of value when compared to the initial condition, providing an optimized condition to the real-world addressed problem. Complementary, the possibility of balancing and sequencing in an integrated mathematical approach allowed to provide a solution that better encompasses the significant differences between the products involved in the considered sequential line.

As suggestions for future research it is mentioned:

- To add to the model the possibility of having production buffers to reduce idle time during sequencing.
- To test different mixes as the reference one on Proposal Analysis 2.
- To allow the change in the quantity of workers/stations according to demand.
- To add auxiliary constraints or to propose pre-processing approaches to aid the model convergence.
- To adapt the model to a Two-sided assembly line balancing problem (TALBP) to better represent practical conditions.
- To create a mathematical model to perform the balancing and sequencing of different mixes simultaneously.
- Apply the methodology and use the mathematical model on different assembly lines of the factory.

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APPENDIX A - TASK TIME

Table 20 – Task time for different models

Task	**Time model 1** (tu)	**Time model 2** (tu)	**Time model 3** (tu)	**Time model 4** (tu)
1	0.00	0.74	0.74	0.74
2	0.00	0.37	0.37	1.25
3	0.00	0.15	0.15	3.90
4	0.00	1.62	0.51	0.74
5	0.00	4.48	0.15	0.15
6	0.00	0.00	8.82	0.00
7	0.00	0.00	2.06	0.00
8	0.00	0.00	2.57	0.00
9	0.00	0.00	0.15	0.00
10	0.00	0.15	0.00	0.15
11	0.00	2.50	0.00	2.50
12	0.00	3.31	0.00	3.31
13	0.00	3.53	0.00	3.53
14	0.00	1.40	0.00	1.40
15	0.00	1.54	0.00	1.54
16	0.00	0.74	0.00	0.74
17	0.00	0.74	0.00	0.74
18	0.00	11.69	0.00	11.69
19	0.00	3.97	0.00	3.97
20	0.00	0.15	0.00	0.15
21	0.00	1.47	0.00	1.47
22	0.00	0.15	0.15	0.15
23	1.47	0.00	0.00	0.00
24	0.74	0.00	0.00	0.00
25	3.45	0.00	0.00	0.00
26	1.32	0.00	0.00	0.00
27	0.66	0.00	0.00	0.00
28	1.10	0.00	0.00	0.00
29	0.74	0.00	0.00	0.00
30	4.70	0.00	0.00	0.00
31	7.42	0.00	0.00	0.00
32	1.54	0.00	0.00	0.00
33	0.51	0.00	0.00	0.00
34	2.43	0.00	0.00	0.00
35	3.60	0.00	0.00	0.00
36	2.43	0.00	0.00	0.00
37	3.38	0.00	0.00	0.00
38	0.96	0.00	0.00	0.00
39	0.00	0.74	10.0	0.74
40	0.00	0.00	1.91	0.00
41	0.00	6.91	0.66	0.66
42	0.00	0.88	0.51	0.88
43	0.00	1.32	1.32	1.32
44	0.00	0.66	0.66	0.66
45	0.00	3.16	0.81	3.16
46	0.00	2.35	0.00	2.35
47	0.00	0.29	0.29	0.29
48	0.00	0.29	0.66	0.29
49	0.00	0.15	1.32	0.15
50	0.00	1.91	0.00	1.91

Source: Own authorship (2023).

Table 20 – Task time for different models (continued)

Task	**Time model 1** (tu)	**Time model 2** (tu)	**Time model 3** (tu)	**Time model 4** (tu)
51	0.00	7.86	0.00	7.86
52	0.00	0.22	0.00	0.22
53	0.00	1.91	0.00	1.91
54	0.00	1.40	0.00	1.40
55	0.00	1.47	0.00	1.47
56	0.00	11.69	0.00	11.69
57	0.00	4.04	0.00	4.04
58	0.00	0.44	0.00	0.44
59	0.00	0.29	0.29	0.29
60	3.53	0.00	0.00	0.00
61	1.32	0.00	0.00	0.00
62	0.66	0.00	0.00	0.00
63	1.10	0.00	0.00	0.00
64	1.32	0.00	0.00	0.00
65	1.25	0.00	0.00	0.00
66	1.03	0.00	0.00	0.00
67	2.79	0.00	0.00	0.00
68	8.01	0.00	0.00	0.00
69	1.54	0.00	0.00	0.00
70	4.19	0.00	0.00	0.00
71	1.69	0.00	0.00	0.00
72	4.85	0.00	0.00	0.00
73	0.22	0.00	0.00	0.00
74	0.00	0.74	0.74	0.74
75	0.00	1.98	1.98	1.98
76	0.00	1.18	1.18	1.18
77	0.00	1.32	1.32	1.32
78	0.00	1.84	1.84	1.84
79	0.00	1.18	1.18	1.18
80	0.00	1.25	1.25	1.25
81	0.00	1.54	1.54	1.54
82	0.00	2.35	2.35	2.35
83	0.00	2.65	2.65	2.65
84	0.00	3.31	3.31	3.31
85	0.00	1.40	1.40	1.40
86	0.00	0.88	0.88	0.88
87	0.00	1.25	1.25	1.25
88	0.00	1.47	1.47	1.47
89	0.00	1.32	1.32	1.32
90	0.00	1.32	1.32	1.32
91	0.00	0.15	0.15	0.15
92	0.00	0.00	0.00	0.00
93	0.00	2.06	2.06	2.06
94	0.00	1.69	1.69	1.69
95	0.00	1.18	1.18	1.18
96	0.00	1.25	1.25	1.25
97	0.00	1.25	1.25	1.25
98	0.00	1.62	1.62	1.62
99	0.00	1.69	1.69	1.69
100	0.00	1.40	1.40	1.40

Source: Own authorship (2023).

Table 20 – Task time for different models (continued)

Task	**Time model 1** (tu)	**Time model 2** (tu)	**Time model 3** (tu)	**Time model 4** (tu)
101	0.00	3.97	3.97	3.97
102	0.00	3.31	3.31	3.31
103	0.00	1.40	1.40	1.40
104	0.00	1.25	1.25	1.25
105	0.00	1.10	1.10	1.10
106	0.00	0.51	0.51	0.51
107	0.00	1.03	1.03	1.03
108	0.00	3.09	0.15	0.15
109	0.00	1.47	1.47	1.47
110	0.00	1.47	0.00	1.47
111	0.00	0.15	1.84	0.15
112	0.00	5.44	0.00	5.44
113	0.00	2.13	0.00	2.13
114	0.00	3.09	0.00	3.09
115	0.00	0.15	0.15	0.15
116	1.76	0.00	0.00	0.00
117	3.60	0.00	0.00	0.00
118	1.03	0.00	0.00	0.00
119	4.92	0.00	0.00	0.00
120	5.73	0.00	0.00	0.00
120	5.73	0.00	0.00	0.00
121	1.03	0.00	0.00	0.00
122	1.62	0.00	0.00	0.00
123	0.44	0.00	0.00	0.00
124	0.00	1.40	1.40	1.40
125	0.00	0.44	0.44	0.44
126	0.00	1.47	1.47	1.47
127	0.00	13.23	0.00	13.23
128	0.00	13.23	0.00	13.23
129	0.00	0.00	1.84	0.00
130	1.84	0.00	0.00	0.00
131	3.31	0.00	0.00	0.00
132	0.74	0.00	0.00	0.00
133	1.03	0.00	0.00	0.00
134	5.73	0.00	0.00	0.00
135	6.62	0.00	0.00	0.00
136	1.54	0.00	0.00	0.00
137	1.54	0.00	0.00	0.00
138	0.44	0.00	0.00	0.00
139	0.00	2.43	2.43	2.43
140	0.00	2.87	2.87	2.87
141	0.00	2.50	2.50	2.50
142	0.00	2.35	2.35	2.35
143	0.00	0.59	0.59	0.59
144	0.00	1.76	1.76	1.76
145	0.00	8.38	8.38	8.38
146	0.00	2.43	2.43	2.43
147	0.00	0.66	0.66	0.66
148	0.00	0.29	0.29	0.29
149	1.18	0.00	0.00	0.00
150	3.01	0.00	0.00	0.00

Source: Own authorship (2023).

Table 20 – Task time for different models (continued)

Task	Time model 1 (tu)	Time model 2 (tu)	Time model 3 (tu)	Time model 4 (tu)
151	1.62	0.00	0.00	0.00
152	2.28	0.00	0.00	0.00
153	2.65	0.00	0.00	0.00
154	1.98	0.00	0.00	0.00
155	2.21	0.00	0.00	0.00
156	3.16	0.00	0.00	0.00
157	1.91	0.00	0.00	0.00
158	0.22	0.00	0.00	0.00
159	0.00	1.25	1.25	1.25
160	0.00	4.56	4.56	4.56
161	0.00	2.35	2.35	2.35
162	0.00	2.28	2.28	2.28
163	0.00	0.51	0.51	0.51
164	0.00	1.54	1.54	1.54
165	0.00	8.09	8.09	8.09
166	0.00	2.28	2.28	2.28
167	0.00	1.84	1.84	1.84
168	0.00	6.39	6.39	6.39
169	0.00	0.96	0.96	0.96
170	0.00	0.44	0.44	0.44
171	0.00	1.25	1.25	1.25
172	0.00	0.00	0.00	0.00
173	0.96	0.00	0.00	0.00
174	2.06	0.00	0.00	0.00
175	1.69	0.00	0.00	0.00
176	2.21	0.00	0.00	0.00
177	2.57	0.00	0.00	0.00
178	1.54	0.00	0.00	0.00
179	7.57	0.00	0.00	0.00
180	1.69	0.00	0.00	0.00
181	0.44	0.00	0.00	0.00
182	0.00	1.54	1.54	1.54
183	0.00	7.86	7.86	7.86
184	0.00	1.62	1.62	1.62
185	0.00	1.54	1.54	1.54
186	0.00	0.37	0.37	0.37
187	0.00	1.25	1.25	1.25
188	0.00	0.22	0.22	0.22
189	0.00	0.37	0.37	0.37
190	0.00	1.25	1.25	1.25

Source: Own authorship (2023).