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ESTABILIZAÇÃO DE SISTEMAS NÃO LINEARES COM SALTOS MARKOVIANOS

DISSERTAÇÃO

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ESTABILIZAÇÃO DE SISTEMAS NÃO LINEARES COM SALTOS MARKOVIANOS

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Orientador: Alessandro do Nascimento Vargas

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por

Sérgio Matsue Filho

Orientador: **Prof. Dr. Alessandro do Nascimento Vargas**

Esta dissertação foi apresentada como requisito parcial à obtenção do grau de MESTRE EM ENGENHARIA ELÉTRICA – Área de Concentração: Sistemas Eletrônicos Industriais, pelo Programa de Pós-Graduação em Engenharia Elétrica – PPGEE – da Universidade Tecnológica Federal do Paraná – UTFPR – Câmpus Cornélio Procópio, às 9h do dia 27 de Outubro de 2018. O trabalho foi aprovado pela Banca Examinadora, composta pelos professores:

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"Don't you tell me what you think that I could be, I'm the one at the sail, I'm the master of my sea". -Believer, Imagine Dragons.

RESUMO

MATSUE FILHO, Sérgio. ESTABILIZAÇÃO DE SISTEMAS NÃO LINEARES COM SALTOS MARKOVIANOS. 30 f. Dissertação – Programa de Pós-graduação em Engenharia Elétrica, Universidade Tecnológica Federal do Paraná. Cornélio Procópio, 2018.

Sistemas sujeitos a saltos Markovianos têm recebido notável atenção nos últimos anos. Por esta razão foram desenvolvidas, neste trabalho, condições que garantem a estabilidade para um certo tipo de sistema não linear sujeito a saltos Markovianos. De modo a ilustrar o resultado teórico desenvolvido, este trabalho apresenta uma aplicação em tempo real para uma válvula automotiva.

Palavras-chave: Estabilidade, Sistemas Sujeitos a saltos markovianos.

ABSTRACT

MATSUE FILHO, Sérgio. STABILIZATION OF NONLINEAR MARKOV JUMP SYSTEMS. 30 f. Dissertação – Programa de Pós-graduação em Engenharia Elétrica, Universidade Tecnológica Federal do Paraná. Cornélio Procópio, 2018.

Markov Jump Systems have received notable attention in last few years. For this reason conditions that guarantee stability for a certain type of nonlinear Markov Jump System were developed in this work. Here is presented a real-time application in an automotive valve, thus illustrating the theoretical result developed.

Keywords: Stability, Markov Jump Systems.

SUMÁRIO

1 INTRODUÇÃO

Sistemas com saltos Markovianos têm recebido consideravel atenção nos últimos anos, devido à sua capacidade de modelar plantas que estão sujeitas a variações abruptas em sua estrutura, seja por falha nos componentes, por mudanças repentinas nos ambiente, etc. Estes sistemas são uma classe especial de sistemas estocásticos, onde cada modo de operação corresponde a um sistema dinâmico, e a transição entre estes modos é regida por uma cadeia de Markov. Alguns exemplos de aplicações podem ser encontrados em economia (COSTA; de Oliveira, 2012), robótica (VARGAS et al., 2013b), controle de potência (UGRINOVSKII; POTA, 2005), e filtragem (WANG et al., 2004; YIN et al., 2011; DONG et al., 2011).

Controle de sistemas não lineares com saltos Markovianos, podem ser encontrados em (WU et al., 2014; LI et al., 2015; YAO; GUO, 2013; ZHA et al., 2017), e estratégias sliding-mode podem ser encontradas em (WU et al., 2010; LI et al., 2016; LIU et al., 2011; WU et al., 2012; LI et al., 2014).

Na literatura, podemos encontrar trabalhos que apresentam condições para estabilização de sistemas não lineares com saltos Markovianos, tais como (KHASMINSKII et al., 2007; ZHANG; BOUKAS, 2009; ZHAO; GUPTA, 2015; VARGAS et al., 2017; COSTA et al., 2013, 2006; LONG; ZHONG, 2017). Aqui, contribuímos para o tópico de estabilidade em segundo momento de sistemas não lineares com saltos Markovianos através de novos resultados que garantem a estabilidade para o sistema não linear abordado.

2 ARTIGO: "STABILIZATION OF MARKOV JUMP SYSTEMS WITH NONLINEAR TERMS"

2.1 INTRODUCTION

In the recent years, special attention has been given to systems subject to Markov-driven events, since they are useful to model dynamical systems whose structure is subject to abrupt random variations. Examples of applications can be found in robotics (VARGAS et al., 2013b), economics (COSTA; de Oliveira, 2012; OLIVEIRA et al., 2009), direct current motors (OLIVEIRA et al., 2014; VARGAS et al., 2015, 2013a; YIN et al., 2014), and filtering (YIN et al., 2011; DONG et al., 2011).

Control of nonlinear Markov jump systems can be found in (WU et al., 2014; LI et al., 2015; YAO; GUO, 2013; ZHA et al., 2017), and sliding-mode strategies can be found in (WU et al., 2010; LI et al., 2016; LIU et al., 2011; WU et al., 2012; LI et al., 2014).

In the literature, we can find works presenting stabilizing conditions for nonlinear Markov jump systems in (KHASMINSKII et al., 2007; ZHANG; BOUKAS, 2009; ZHAO; GUPTA, 2015; VARGAS et al., 2017; COSTA et al., 2013, 2006; LONG; ZHONG, 2017). Here, we contribute to the topic of second moment stability of nonlinear Markov jump systems through novel results that guarantee such stability for the underlying nonlinear stochastic system.

We explore in this work the following Markov jump system. Let

$$x(t) = [x_1(t) \quad x_2(t)]' \in \mathbb{R}^n,$$

be the system state, in which $x_1(t)$ and $x_2(t)$ represent partitions. The system under study reads as

$$
\mathrm{d}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} = \left(\begin{bmatrix} A_{11,\theta(t)} & A_{12,\theta(t)} \\ A_{21,\theta(t)} & A_{22,\theta(t)} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} g_{\theta(t),1}(x(t)) \\ \vdots \\ g_{\theta(t),q}(x(t)) \\ g_{\theta(t),q+1}(x(t)) \\ \vdots \\ g_{\theta(t),n}(x(t)) \end{bmatrix} \right) \mathrm{d}t
$$

$$
+ \begin{bmatrix} \emptyset_q \\ B_{\theta(t)} \end{bmatrix} (\psi(x(t)) + \gamma(x(t))u(t))\,\mathrm{d}t + H_{\theta(t)}\mathrm{d}w(t), \tag{2.1}
$$

for all $t \geq 0$, $x(0) = x_0 \in \mathbb{R}^n$, where $w(t)$ denotes a standard r-dimensional Brownian motion, and $\{\theta(t)\}$ denotes an irreducible continuous-time Markov process having $S = \{1, \ldots, N\}$ as state space, $\emptyset_q$ represents a null-square q-dimensional matrix, and $\gamma(x)$ and $\psi(x)$ represent nonlinear functions, where $\gamma(x)$ is non-singular. The functionals $g_{\theta(t),\ell}(x(t))$, $\ell = 1, \ldots, N$ are given. We assume that $x(0)$, $w(t)$, and $\theta(t)$ are mutually independent random variables for each $t \geq 0$. The values of the matrices $A_{11,\theta(t)}, A_{12,\theta(t)}, A_{21,\theta(t)}, A_{22,\theta(t)}, B_{\theta(t)}, H_{\theta(t)}$ are given, and $B_{\theta(t)}$ admits a Moore-Penrose inverse matrix.

Inspired on the state-feedback linearization method, which is an useful technique that converts a nonlinear state equation into a linear equation [(KHALIL, 2002),pg. 506], we purposefully define the controller $u(t)$ as

$$
u(t) = \gamma^{-1}(x(t)) \left[-\psi(x(t)) + K_{\theta(t)}x(t) - B^{+}_{\theta(t)} \begin{bmatrix} g_{\theta(t),q+1}(x(t)) \\ \vdots \\ g_{\theta(t),q+s}(x(t)) \end{bmatrix} \right], \tag{2.2}
$$

where $B^{+}_{\theta(t)}$ represents the Moore-Penrose inverse of $B_{\theta(t)}$.

Substituting the controller (2.2) into (2.1), we obtain

$$
\mathrm{d}x(t) = \left(\tilde{A}_{\theta(t)}x(t) + \begin{bmatrix} g_{\theta(t),1}(x(t)) \\ \vdots \\ g_{\theta(t),q}(x(t)) \\ \emptyset \end{bmatrix} \right) \mathrm{d}t + H_{\theta(t)}\mathrm{d}w(t), \quad \forall t \geq 0,\ x(0) = x_0 \in \mathbb{R}^n, \tag{2.3}
$$

where

$$\tilde{A}_i := \begin{bmatrix} A_{11,\theta(t)} & A_{12,\theta(t)} \\ A_{21,\theta(t)} + B_{\theta(t)}K_{\theta(t),1} & A_{22,\theta(t)} + B_{\theta(t)}K_{\theta(t),2} \end{bmatrix}. \tag{2.4}$$

From now on, we restrict our study to the controlled system (2.3) subject to (2.4).

Remark 2.1.1. *Systems (2.1) and (2.3) are equivalent only if (2.2) is used. In the sequence, we derive conditions to compute $K_{\theta(t),1}$ and $K_{\theta(t),2}$ in (2.4) in a way that (2.3) becomes second moment stable, a concept defined next.*

Definition 2.1.1. *((ARNOLD, 1974, Defn. 11.3.1, p. 188)(VARGAS et al., 2017, Defn. 1.1, p. 333)). The nonlinear Markov jump system in* (2.3) *is second moment stable if there exists some constant $c = c(x_0)$ such that*

$$\mathrm{E}[\|x(t)\|^2] \leq c, \quad \forall t \geq 0.$$

Consider now the elements of the vector $x(t)$ written explicitly in the form $x(t) \equiv [x_{[1]}(t), \ldots, x_{[n]}(t)]'$.

Assumption 2.1.1. [*Adapted from (MAO, 2007, p.62, Cor.4.5)*]. *There exist two constant $\alpha_1, \alpha_2 > 0$ such that*

$$x_{[\ell]} g_{i,\ell}(x) \leq \alpha_2 + \alpha_1 x_{[\ell]}^2, \quad \forall x \in \mathbb{R}^n, \quad i = 1, \ldots, N, \quad \ell = 1, \ldots, q. \tag{2.5}$$

Remark 2.1.2. *Assumption 2.1.1 is known in the literature as linear growth, and it is used frequently,(LI et al., 2017).*

2.2 NOTATIONS AND AUXILIARY RESULTS

The n-dimensional Euclidean space is denoted by $\mathbb{R}^n$, and its corresponding norm is represented by $\|.\|$. The trace operator is represented by $\mathbf{tr}\{.\}$. The notation of the identity matrix on $\mathbb{R}^{n\times n}$ is I_n. The symbol $\mathbb{1}_{\mathscr{C}}$ means the Dirac measure of $\mathscr{C}$, i.e., if condition $\mathscr{C}$ is true, then $\mathbb{1}_{\mathscr{C}}$ is equal to 1, otherwise it is equal to 0. Given two matrices $U \in \mathbb{R}^{n\times n}$ and $V \in \mathbb{R}^{m\times m}$, the square diagonal matrix made up by V and U is represented by $\mathbf{diag}(V,U)$. In case of $\mathbf{diag}(V_1, \ldots V_N)$, we set $\mathbf{diag}(V_i)_{\{i=1,\ldots,N\}}$.

The homogeneous process $\theta = \{(\theta_t, \mathscr{F}, t \in \mathbb{R}^+)\}$ is right continuous and takes values on the set S. We assume also that

$$P(\theta_{t+h=j}|\theta_t = i) = \begin{cases} \pi_{ij}h + o(h), & i \neq j, \\ 1 + \pi_{ij}h + o(h), & i = j, \end{cases} \tag{2.6}$$

where $\left[(\pi_{ij})\right]$ is the stationary infinite-dimensional transition rate matrix of $\{\theta\}$ with $0 \leq \pi_{ij}$, $i \neq j$, and $0 \leq \pi_i := -\pi_{ii} = \sum_{\{j: j \neq i\}} \pi_{ij} \leq \rho$ for all $i \in \mathscr{S}$, $o(h)$ means an infinitesimal of higher order than h, i.e., $\lim_{h \downarrow 0} \dfrac{o(h)}{h} = 0$. Denote $p_i(t) := \mathbb{P}(\theta_t = i)$ for any $i \in S$.

The notation $\mathbb{S}^m$ will be used to refer to the set of symmetric positive-definite matrices of dimension n.

Lemma 2.2.1. *(**Projection Lemma** (PIPELEERS et al., 2009, p.511)). Given a symmetric matrix $-Z \in \mathbb{S}^m$, and two arbitrary matrices U and V of column dimension m, then there exists an unstructured matrix X that satisfies*

$$U'XV + V'X'U + Z < 0, \tag{2.7}$$

if and only if the following projection inequalities with respect to X are satisfied:

$$N'_U Z N_U < 0, \tag{2.8}$$

$$N'_V Z N_V < 0, \tag{2.9}$$

where N_U and N_V are arbitrary matrices whose columns form a basis for the null spaces of U and V, respectively.

2.2.1 PRELIMINARY RESULTS FOR AUTONOMOUS MARKOV JUMP LINEAR SYSTEM

Consider the autonomous Markov jump linear system

$$\dot{z}(t) = A_{\theta(t)} z(t), \quad t \geq 0, \quad \theta(0) = \theta_0, \quad z(0) = z_0. \tag{2.10}$$

Definition 2.2.1. *((COSTA et al., 2013, Defn. 3.2 p.36)(FRAGOSO; COSTA, 2005b, Defn. 3.2, p.1173)). We say the system in (2.10) is mean-square stable if* $\mathrm{E}\left[\|z(t)\|^2\right] \to 0$ *as $t \to +\infty$ for all $z_0 \in \mathbb{R}^n$.*

The following result presents an equivalent condition to check the mean-square stability of (2.10).

Proposition 2.2.1. *((COSTA et al., 2013, Thm. 3.21 p.48)(FRAGOSO; COSTA, 2005a, Thm. 4.19, p.1179)). The system in (2.10) is mean-square stable if and only if there exist matrices P_i's on $\mathbb{S}^n$ such that*

$$A_iP_i + P_iA_i' + \sum_{j\in S} \pi_{ji}P_j < 0, \quad i = 1,\ldots,N.$$

The next result will be useful in the proof of the main result of this paper.

Lemma 2.2.2. *The system in (2.10) is mean square stable if there exist matrices P_i's on $\mathbb{S}^n$ and X_i's on $\mathbb{R}^{n\times m}$ such that*

$$\begin{bmatrix} A_iX_i + X_i'A_i' & P_i + A_iX_i - X_i' & R_i \\ P_i + X_i'A_i' - X_i & -X_i - X_i' & -X_i \\ R_i & -X_i' & A_iX_i + X_i'A_i' - R_i \end{bmatrix} < 0, \quad i = 1,\ldots,N, \tag{2.11}$$

where $R_i = \sum_{j=1}^{N} \pi_{ji}P_j$.

Proof of Lemma 2.2.2. Set $Z \in \mathbb{R}^{3n\times 3n}$ as

$$Z = \begin{bmatrix} 0 & P_i & R_i \\ P_i & 0 & 0 \\ R_i & 0 & -R_i \end{bmatrix}. \tag{2.12}$$

Define $N_U = \begin{bmatrix} I \\ A_i' \\ I \end{bmatrix}$ and $N_V = \begin{bmatrix} -I \\ I \\ 0 \end{bmatrix}$. It follows that

$$\begin{aligned} N_U'ZN_U &= \begin{bmatrix} I & A_i & I \end{bmatrix} \begin{bmatrix} 0 & P_i & R_i \\ P_i & 0 & 0 \\ R_i & 0 & -R_i \end{bmatrix} \begin{bmatrix} I \\ A_i' \\ I \end{bmatrix} \\ &= A_iP_i + P_iA_i' + R_i, \end{aligned} \tag{2.13}$$

and that

$$N_V'ZN_V = \begin{bmatrix} -I & I & 0 \end{bmatrix} \begin{bmatrix} 0 & P_i & R_i \\ P_i & 0 & 0 \\ R_i & 0 & -R_i \end{bmatrix} \begin{bmatrix} -I \\ I \\ 0 \end{bmatrix} = -2P_i < 0. \tag{2.14}$$

Note that (2.13) holds if and only if (2.10) is mean square stable (see Prop. 2.2.1). Now, define U and V so that the columns of N_U and N_V form a basis for the null space of U and V; for instance, set $U = \begin{bmatrix} A_i' & -I & 0 \\ 0 & -I & A_i' \end{bmatrix}$ so that

$$UN_U = \begin{bmatrix} A_i' & -I & 0 \\ 0 & -I & A_i' \end{bmatrix} \begin{bmatrix} I \\ A_i' \\ I \end{bmatrix} = 0; \tag{2.15}$$

and $V = \begin{bmatrix} I & I & 0 \\ 0 & 0 & I \end{bmatrix}$ to obtain

$$VN_V = \begin{bmatrix} I & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{bmatrix} -I \\ I \\ 0 \end{bmatrix} = 0. \tag{2.16}$$

Take $X_{11,i} = X_{22,i} = X_i$, and $X_{12,i} = X_{21,i} = \emptyset$. If the next inequality holds

$$\begin{aligned} &\begin{bmatrix} 0 & P_i & R_i \\ P_i & 0 & 0 \\ R_i & 0 & -R_i \end{bmatrix} + \begin{bmatrix} A_i & 0 \\ -I & -I \\ 0 & A_i \end{bmatrix} \begin{bmatrix} X_{11,i} & X_{12,i} \\ X_{21,i} & X_{22,i} \end{bmatrix} \begin{bmatrix} I & I & 0 \\ 0 & 0 & I \end{bmatrix} + \begin{bmatrix} I & 0 \\ I & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} X_{11,i}' & X_{21,i}' \\ X_{12,i}' & X_{22,i}' \end{bmatrix} \begin{bmatrix} A_i' & -I & 0 \\ 0 & -I & A_i' \end{bmatrix} \\ &= \begin{bmatrix} A_iX_i + X_i'A_i' & P_i + A_iX_i - X_i' & R_i \\ P_i + X_i'A_i' - X_i & -X_i - X_i' & -X_i \\ R_i & -X_i' & A_iX_i + X_i'A_i' - R_i \end{bmatrix} < 0, \end{aligned} \tag{2.17}$$

then Lemma 2.2.1 implies that $N_U'ZN_U$ and from (2.13) $A_iP_i + P_iA_i' + R_i < 0$, and Proposition 2.2.1 completes the proof. □

2.3 MAIN RESULT

Consider the second moment matrix

$$\mathscr{X}_i(t) = \mathrm{E}[x(t)x(t)' \mathbb{1}_{\theta(t)=i}], \quad i = 1, \ldots, N, \quad \forall t \geq 0. \tag{2.18}$$

Consider also the symmetric, positive semi-definite matrix $V_i(t)$ on $\mathbb{S}^m$, solution of the matrix differential equation (and α_1 and α_2 as in Assumption 2.1.1)

$$\begin{aligned} \dot{V}_i(t) &= V_i(t)\left(\tilde{A}_i+\alpha_1 I\right)' + \left(\tilde{A}_i+\alpha_1 I\right)V_i(t) \\ &+ \sum_{j=1}^{N}\pi_{ji}V_j(t)+H_iH_i'p_i(t)+2\alpha_2 I, \quad i=1,\ldots,N, \quad \forall t\geq t_0, \quad V_i(t_0)\in\mathbb{S}^n \end{aligned} \tag{2.19}$$

where the closed-loop matrices $\tilde{A}_i$, $i=1,\ldots,N$, satisfy (2.4).

Theorem 2.3.1. *There exists some $t_0 \geq 0$ such that*

$$\mathrm{tr}\{\mathscr{X}_i(t)\} \leq \mathrm{tr}\{V_i(t)\}, \quad i=1,\ldots,N, \quad \forall t\geq t_0, \tag{2.20}$$

where $V(t)$ satisfies (2.19) *with $V(t_0)=\mathscr{X}(t_0)$.*

The proof of Theorem 2.3.1 can be found in Appendix.

Remark 2.3.1. *Provided that $V(t)$ is uniformly bounded, then $\mathrm{E}[\|x(t)\|^2]=\mathrm{tr}\{\mathrm{E}[x(t)x(t)']\}=\sum_{i=1}^{N}\mathrm{tr}\{\mathscr{X}_i(t)\}$ is bounded because it follows from Theorem 2.3.1.*

Now, we are able to present the main result of this paper.

Theorem 2.3.2. *If there exist unstructured matrices $X_{1,i},X_{2,i},Y_{1,i},Y_{2,i}$, and matrices P_i's on $\mathbb{S}^n$ such that*

$$\begin{bmatrix} M_i+M_i'+\alpha_1(X_i+X_i') & P_i+M_i+\alpha_1 X_i-X_i' & R_i \\ P_i+M_i'+X_i'\alpha_1-X_i & -X_i-X_i' & -X_i \\ R_i & -X_i' & M_i+M_i'+\alpha_1(X_i+X_i')-R_i \end{bmatrix} < 0, \tag{2.21}$$

i=1,...,N, with $X_i=\mathrm{diag}(X_{1,i},X_{2,i})$, then the autonomous Markov jump linear system

$$\dot{z}(t)=\left(\tilde{A}_{\theta(t)}+\alpha_1 I\right)z(t), \quad z(0)=z_0, \tag{2.22}$$

is mean square stable, where M_i is defined below:

$$M_i = \begin{bmatrix} A_{11,i}X_{1,i} & A_{12,i}X_{2,i} \\ A_{21,i}X_{1,i}+B_iY_{1,i} & A_{22,i}X_{2,i}+B_iY_{2,i} \end{bmatrix}. \tag{2.23}$$

The control gain K_i, $i=1,\ldots,N$, required in (2.2), and also in (2.4), is given by

$$K_i=\begin{bmatrix} Y_{1,i}X_{1,i}^{-1} & Y_{2,i}X_{2,i}^{-1}\end{bmatrix}. \tag{2.24}$$

Proof of Theorem 2.3.2.

Consider

$$\tilde{A}_i X_i = \begin{bmatrix} A_{11,i}X_{1,i} & A_{12,i}X_{2,i} \\ A_{21,i}X_{1,i} + B_i Y_{1,i} & A_{22,i}X_{2,i} + B_i Y_{2,i} \end{bmatrix}, \quad i = 1,\ldots,N. \tag{2.25}$$

Setting $Y_{1,i} = K_{1,i}X_{1,i}$ and $Y_{2,i} = K_{2,i}X_{2,i}$, we obtain

$$\tilde{A}_i X_i = \begin{bmatrix} A_{11,i}X_{1,i} & A_{12,i}X_{2,i} \\ (A_{21,i} + B_i K_{1,i})X_{1,i} & (A_{22,i} + B_i K_{2,i})X_{2,i} \end{bmatrix}. \tag{2.26}$$

Substituting (2.26) into (2.21), we obtain

$$\begin{bmatrix} (\tilde{A}_i + \alpha_1 I)X_i + X_i'(\tilde{A}_i' + \alpha_1 I) & P_i + (\tilde{A}_i + \alpha_1 I)X_i - X_i' & R_i \\ P_i + X_i'(\tilde{A}_i' + \alpha_1 I) - X_i & -X_i - X_i' & -X_i \\ R_i & -X_i' & (\tilde{A}_i + \alpha_1 I)X_i + X_i'(\tilde{A}_i' + \alpha_1 I) - R_i \end{bmatrix} < 0, \tag{2.27}$$

and using (2.27) and Lemma 2.2.2, we conclude that

$$\dot{z}(t) = (\tilde{A}_{\theta(t)} + \alpha_1 I)z(t), \tag{2.28}$$

is mean-square stable. □ Let us now recall an important result form the literature.

Proposition 2.3.1. *((FRAGOSO; COSTA, 2005a, Thm 5.6), (COSTA et al., 2013, Thm. 3.25, p. 52),(VARGAS et al., 2017, Prop. 3.1, p. 335)). If the system in (2.22) is mean-square stable, then the limit* $\lim_{t\to\infty} V(t)$ *in (2.19) does exist and it does not depend on the initial condition* $V(t_0) \in \mathbb{R}^{n\times n}$.

The existence of $\lim_{t\to\infty} V(t)$ from Proposition 2.3.1 assures that $V(t)$ is uniformly bounded; this fact, Theorem 2.3.1, and Remark 2.3.1 allow us to conclude that (2.1) is second moment stable. This assertion is summarized in the next corollary.

Corollary 2.3.1. *Under the conditions of Theorem 2.3.2, the nonlinear Markov jump system in (2.1) is second moment stable.*

2.3.1 CONTROL WITH NO MODE OBSERVATION

In this section, we are interested in the control law

$$u(t) = Kx(t), \forall t \geq 0. \tag{2.29}$$

Note that K in (2.29) does not depend on $\theta(t)$, in contrast with (2.2). The next result is immediate from Theorem 2.3.2.

Corollary 2.3.2. *If there exist unstructured matrices* X_1, X_2, Y_1, Y_2*, and symmetric positive-definite matrix* P_i*'s on* $\mathbb{S}^n$ *such that*

$$\begin{bmatrix} M_i + M_i' + \alpha_1 X + X'\alpha_1 & P_i + M_i + \alpha_1 X - X' & R_i \\ P_i + M_i' + X'\alpha_1 - X & -X - X' & -X \\ R_i & -X' & M_i + M_i' + \alpha_1 X + X'\alpha_1 - R_i \end{bmatrix} < 0, \quad (2.30)$$

with $X = \mathrm{diag}(X_1, X_2)$*, then the autonomous Markov jump linear system*

$$\dot{z}(t) = \left(\tilde{A}_{\theta(t)} + \alpha_1 I\right) z(t), \quad z(0) = z_0, \quad (2.31)$$

is mean square stable, where M *is defined below:*

$$M_i = \begin{bmatrix} A_{11,i} X_1 & A_{12,i} X_2 \\ A_{21,i} X_1 + B_i Y_1 & A_{22,i} X_2 + B_i Y_2 \end{bmatrix}.$$

The control gain K*, required in (2.2), and also in (2.4), is given by*

$$K = \begin{bmatrix} Y_1 X_1^{-1} & Y_2 X_2^{-1} \end{bmatrix}. \quad (2.32)$$

2.4 APPLICATION: CONTROL OF AN AUTOMOTIVE THROTTLE VALVE

Practical experiments were carried out to illustrate our main theoretical findings. The experiments were based on a real-time automotive valve referred to as throttle.

The automotive throttle valve consists of a circular plate that moves around a central axis. It regulates the power produced by spark-ignition engines, through controlling the amount of air that flows into the combustion chamber.

The experiments were conducted in a laboratory testbed (see Figure 2.4.0.1), composed by the following equipments: a unity of Quanser Q4 real-time control board that allowed us to collect real-time data with MATLAB/Simulink software; the Quanser Q4 board data sampling was configured to 1 ms; a unity of Quanser VoltPAQ-X1 power amplifier to supply the electrical current and voltage consumed by the throttle; and a four-years-old unity of Throttle Body code number 036133062A removed from a crashed Volkswagen Golf 1.6.

The throttle is assembled with an internal sensor of position, which measures

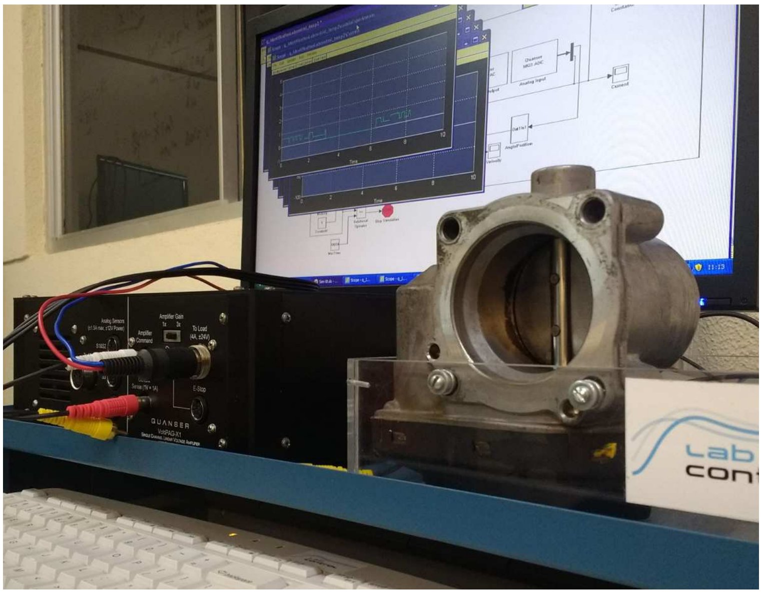


Figure 2.4.0.1: Laboratory testbed.

the range of operation of the throttle plate from 0 to 90 degrees, in equivalence to 0 to 5 V, respectively. The velocity is computed by a first-order numerical derivative of the valve's position. The electric power consumed by the throttle was measured by a current sensor. For convenience, we denote the position, velocity, electrical current, and input voltage by $x_{[1]}(t)$,$x_{[2]}(t)$,$x_{[3]}(t)$, and $u(t)$, respectively.

2.4.1 MODELING AND IDENTIFICATION

The model used here is based on the traditional, physically driven continuous-time linear model (VASAK et al., 2007; MORARI et al., 2003):

$$\begin{bmatrix} \dot{x}_{[1]}(t) \\ \dot{x}_{[2]}(t) \\ \dot{x}_{[3]}(t) \end{bmatrix} = \begin{bmatrix} 0 & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x_{[1]}(t) \\ x_{[2]}(t) \\ x_{[3]}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix} u(t) \tag{2.33}$$

In order to identify the parameters $a_{12},\ldots,b$, pseudo-random-signals were applied as input of the throttle device in an open-loop configuration during 20 seconds, which resulted in 20.000 data points. The collected data for position and velocity, were used to estimate the parameters of (2.33) through minimizing the corresponding mean-square error, see Figure 2.4.1.1.

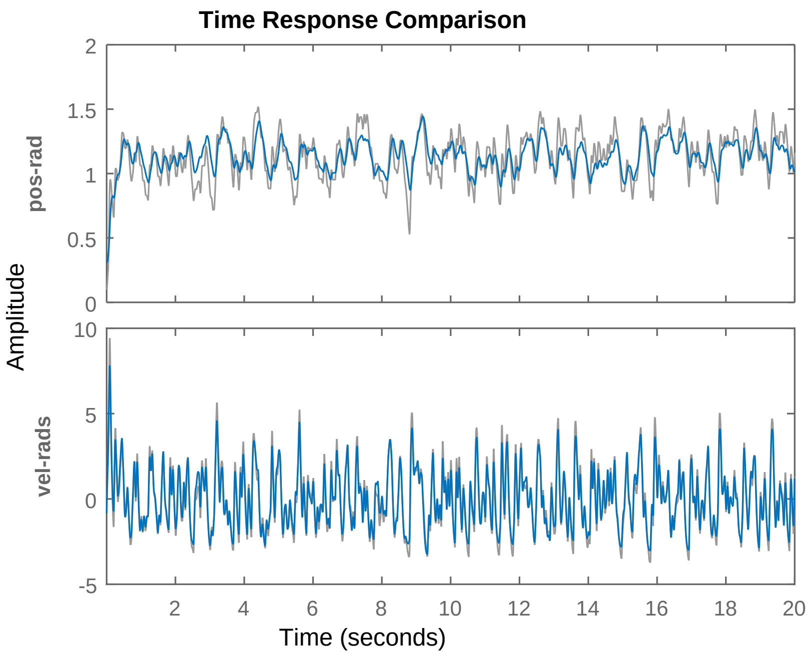


Figure 2.4.1.1: Simulated and real-time data.

2.4.1.1 MODELING THE THROTTLE AS A MARKOV JUMP SYSTEM WITH NONLINEAR TERMS

Two operation modes were studied. The first mode, i.e, $\theta(t) = 1$, represented the valve working on perfect conditions; and the second mode, i.e, $\theta(t) = 2$, represented a failure, which imposed. The laboratory was arranged so as to impose the valve working at 25% of its input nominal voltage. In the failure mode, we imposed the throttle velocity be corrupted by the square root of the current, made for sake of illustration. This constraint justifies the rightmost term of the system below.

$$\begin{bmatrix} \dot{x_{[1]}}(t) \\ \dot{x_{[2]}}(t) \\ \dot{x_{[3]}}(t) \end{bmatrix} = \begin{bmatrix} 0 & a_{12,\theta(t)} & 0 \\ a_{21,\theta(t)} & a_{22,\theta(t)} & a_{23,\theta(t)} \\ 0 & a_{32,\theta(t)} & a_{33,\theta(t)} \end{bmatrix} \begin{bmatrix} x_{[1]}(t) \\ x_{[2]}(t) \\ x_{[1]}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ b_{\theta(t)} \end{bmatrix} u(t) + \begin{bmatrix} |x_{[3]}(t)|^{\frac{1}{2}} \\ 0 \\ 0 \end{bmatrix}. \quad (2.34)$$

The parameters of (2.34) are shown in Table 2.4.1.1.

Parameters	Mode 1 ($\theta(t)=1$)	Mode 2 ($\theta(t)=2$)
$a_{12,\theta(t)}$	0.8106	0.8106
$a_{21,\theta(t)}$	-58.08	-58.08
$a_{22,\theta(t)}$	-21.11	-21.11
$a_{23,\theta(t)}$	-7019	-7019
$a_{33,\theta(t)}$	-64.34	-64.34
$b_{\theta(t)}$	-0.5815	-0.145375

Table 2.4.1.1: Parameters of the system in (2.34).

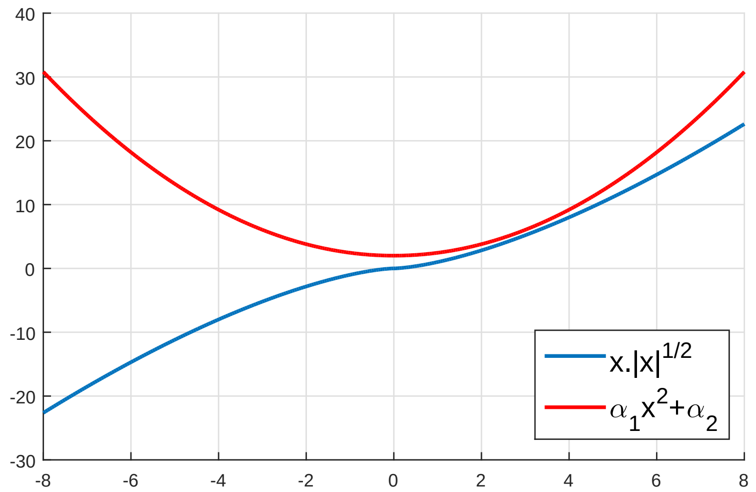


Figure 2.4.2.1: Verification of values that satisfy the Assumption 2.1.1

2.4.2 CONTROL DESIGN

As can be seen in Figure 2.4.2, Assumption 2.1.1 is satisfied when $\alpha_1 = 0.45$ and $\alpha_2 = 2$, i. e.,

$$0.45x^2 + 2 \geq x|x|^{1/2}.$$

The purpose of this section is to illustrate state-feedback with no Markov jump observation, the scheme shown in (2.29). For this purpose, we computed the LMIs of Corolary 2.3.2 with the next probability matrix:

$$\pi = \begin{bmatrix} -5 & 5 \\ 5 & -5 \end{bmatrix}. \tag{2.35}$$

A gain that turns the LMIs in (2.3.2) feasible is

$$K = \begin{bmatrix} -0.5 & -0.005 & -0.5 \end{bmatrix}. \tag{2.36}$$

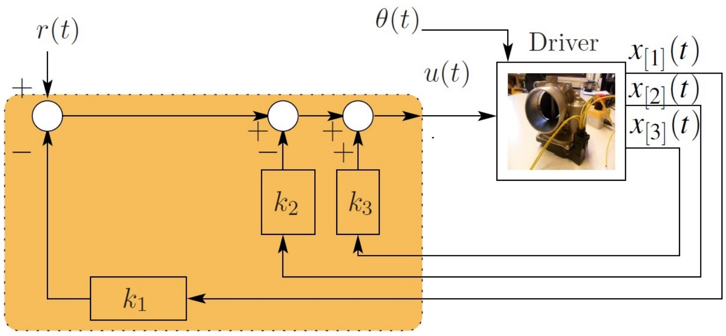


Figure 2.4.2.2: Model configuration.

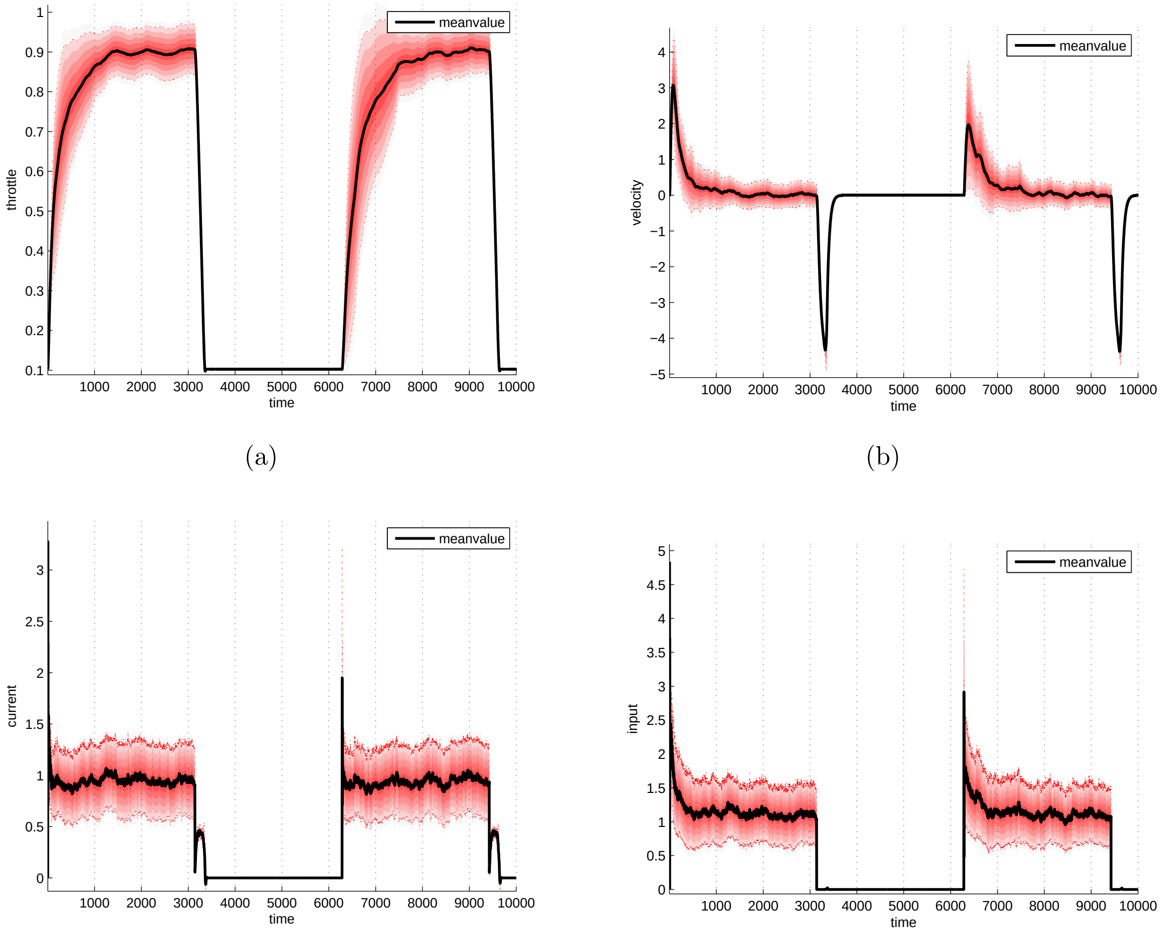
This gain was implemented in the laboratory testbed working under normal and failure modes, driven by Markov jumps, with reference signal r_k switching either at $r_k = 1.41$ or $r_k = 0$.

Figure 2.4.2.3 summarizes the mean and standard deviation of trajectories for 100 distinct evaluations. As can be seen, the system showed a stable behavior, which confirms the results of Theorem 2.3.2 and Corollaries 2.3.1 and 2.3.2.

2.5 CONCLUDING REMARKS

The main contribution of this work is the relatively easy-to-check LMI-conditions of Theorem 2.3.2 that guarantee the second moment stability of a certain type of non-linear Markov jump systems.

Furthermore this work presents data from real-time experiments, designed to control an automotive throttle valve in practice. Experimental data support the potential of our approach for applications.



(a) (b) (c) (d)

Figure 2.4.2.3: Mean and standard deviation

3 APPENDIX - PROOF OF THEOREM 2.3.1

Let us define the operator $\mathscr{G}_i : \mathbb{R}^n \to \mathbb{R}^n$, $i = 1, \ldots, N$, as

$$\mathscr{G}_i(x) = \begin{bmatrix} g_{i,1}(x) \\ \vdots \\ g_{i,q}(x) \\ \emptyset \end{bmatrix}, \forall x \in \mathbb{R}^n. \tag{3.1}$$

In the next result, $o(h)$ means an infinitesimal of higher order than h, i.e., $\lim_{h\downarrow 0} o(h)/h$ equals zero.

Proposition 3.0.1. *((FRAGOSO; COSTA, 2005a, Lem. 4.2), (COSTA et al., 2013, Ch. 3)). Assume that $c(t)$ on $\mathbb{R}^{n\times m}$ is $\mathscr{F}_t$-measurable and that $c_i(t) := \mathrm{E}[c(t)\mathbb{1}_{\theta(t)=i}]$ exists. Then $\mathrm{E}[c(t)\mathrm{d}(\mathbb{1}_{\theta(t)=i})] = \sum_{j=1}^{N} \pi_{ji} c_j(t)\mathrm{d}t + o(\mathrm{d}t)$.*

Using the Itô's rule in the term $\mathscr{X}_i(t) = \mathrm{E}\left[x(t)x(t)'\mathbb{1}_{\theta(t)=i}\right]$, we can write $d\mathscr{X}_i(t)$ as (e.g.(COSTA et al., 2013, Prop. 3.28, p. 56))

$$\begin{aligned} d\mathrm{E}\left[x(t)x(t)'\mathbb{1}_{\theta(t)=i}\right] &= \mathrm{E}\left[dx(t)dx(t)'\mathbb{1}_{\theta(t)=i}\right] + \mathrm{E}\left[dx(t)x(t)'\mathbb{1}_{\theta(t)=i}\right] \\ &+ \mathrm{E}\left[x(t)dx(t)'\mathbb{1}_{\theta(t)=i}\right] + \mathrm{E}\left[x(t)x(t)'d\left(\mathbb{1}_{\theta(t)=i}\right)\right]. \end{aligned} \tag{3.2}$$

Next, each term in the right-hand side of (3.2) is evaluated. The first term can be developed as follows:

$$\mathrm{E}\left[dx(t)dx(t)'\mathbb{1}_{\theta(t)=i}\right] = \mathrm{E}\left[H_{\theta(t)}dw(t)dw(t)'H_{\theta(t)}\mathbb{1}_{\theta(t)=i}\right] = H_i H_i' p_i(t)dt. \tag{3.3}$$

The second term in the right-hand side of (3.2) is identical to (using notation from (2.18))

$$\begin{aligned} \mathrm{E}\left[dx(t)x(t)'\mathbb{1}_{\theta(t)=i}\right] &= \mathrm{E}\left[\left(\tilde{A}_{\theta(t)}x(t) + \mathscr{G}_{\theta(t)}\left(x(t)\right)\right)x(t)'\mathbb{1}_{\theta(t)=i}\right]dt \\ &+ \mathrm{E}\left[H_\theta(t)dw(t)x(t)'\mathbb{1}_{\theta(t)=i}\right] \\ &= \tilde{A}_i\mathscr{X}_i(t)dt + \mathrm{E}\left[\mathscr{G}_{\theta(t)}\left(x(t)\right)x(t)'\mathbb{1}_{\theta(t)=i}\right]dt. \end{aligned} \tag{3.4}$$

The last term in the right-hand side of (3.2) follows from Proposition 3.0.1, which gives

$$\mathrm{E}\left[x(t)x(t)'d(\mathbb{1}_{\theta(t)=i})\right] = \sum_{j=1}^{N} \pi_{ji}\mathscr{X}_j(t)dt + o(dt). \tag{3.5}$$

Combining (3.2)-(3.5) yields

$$\begin{aligned}\dot{\mathscr{X}_i} &= \mathscr{X}_i(t)\tilde{A}_i' + \tilde{A}\mathscr{X}_i(t) + \sum_{j=1}^{N} \pi_{ji}\mathscr{X}_j(t) + H_i H_i' p_i(t) \\ &\quad + \mathrm{E}\left[x(t)\mathscr{G}_{\theta(t)}\left(x(t)\right)' \mathbb{1}_{\theta(t)=i}\right] + \mathrm{E}\left[\mathscr{G}_{\theta(t)}\left(x(t)\right)x(t)' \mathbb{1}_{\theta(t)=i}\right].\end{aligned} \tag{3.6}$$

Applying the trace operator in the rightmost element of (3.6), we obtain

$$\begin{aligned}&\mathrm{tr}\left\{E\left[\begin{bmatrix} g_{\theta(t),1}(x(t)) \\ \vdots \\ g_{\theta(t),q}(x(t)) \\ \emptyset \end{bmatrix}\left[x_{[1]}(t),\ldots,x_{[n]}(t)\right]\mathbb{1}_{\theta(t)=i}\right]\right\} \\ &= \mathrm{E}\left[\mathrm{tr}\left\{\begin{bmatrix} g_{\theta(t),1}(x(t)) \\ \vdots \\ g_{\theta(t),q}(x(t)) \\ \emptyset \end{bmatrix}\left[x_{[1]}(t),\ldots,x_{[n]}(t)\right]\mathbb{1}_{\theta(t)=i}\right\}\right] \\ &= \mathrm{E}\left[\mathrm{tr}\left\{\left[\begin{array}{cccc|c} g_{\theta(t),1}x_{[1]}(x(t)) & 0 & \ldots & 0 & 0 \\ 0 & g_{\theta(t),2}x_{[1]}(x(t)) & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \ldots & g_{\theta(t),q}x_{[1]}(x(t)) & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \end{array}\right]\mathbb{1}_{\theta(t)=i}\right\}\right].\end{aligned} \tag{3.7}$$

Using Assumption 2.1.1 in (3.7), we have

$$\mathrm{tr}\left\{\mathrm{E}\left[\mathscr{G}_{\theta(t)}\left(x(t)\right)x(t)'\mathbb{1}_{\theta(t)=i}\right]\right\} \le \mathrm{E}\left[\mathrm{tr}\left\{\alpha\begin{bmatrix} x_{[1]}^2 & \ldots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \ldots & x_{[q]}^2 \end{bmatrix}\mathbb{1}_{\theta(t)=i} + 2\alpha_2 I\right\}\right]. \tag{3.8}$$

The inequality in (3.8) results in

$$\mathrm{tr}\left\{E\left[\mathscr{G}_{\theta(t)}\left(x(t)\right)x(t)'\mathbb{1}_{\theta(t)=i}\right]\right\} \le \mathrm{tr}\left\{\alpha_1\mathscr{X}_i(t) + 2\alpha_2 I\right\}. \tag{3.9}$$

Now, by using the trace operator on both sides of (3.6), together with (3.9), we obtain

$$\begin{aligned}\operatorname{tr}\left\{\dot{\mathscr{X}_i}(t)\right\} \;\le\; & \operatorname{tr}\Big\{\mathscr{X}_i(t)\left(\tilde{A}_i+\alpha_1 I\right)'+\left(\tilde{A}_i+\alpha_1 I\right)\mathscr{X}_i(t) \\ & +\sum_{j=1}^{N}\pi_{ji}\mathscr{X}_j(t)+H_iH_i'p_i(t)+2\alpha_2 I\Big\}. \end{aligned} \tag{3.10}$$

With $V(t)$ as in (2.19), $V(t_0)=X(t_0)$, we obtain $\operatorname{tr}\{X_i(t)\}\le\operatorname{tr}\{V_i(t)\}$, for $i=1,\ldots,N$, since $\varepsilon>0$ arbitrarily taken . The result, then follows from (2.19) and (3.10).

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